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Event-based average consensus of disturbed MASs via fully distributed sliding mode control

Deyin Yao, *Member, IEEE*, Hongyi Li, *Senior Member, IEEE*, and Yang Shi, *Fellow, IEEE*

***Abstract*—Under undirected graph, we design the fully distributed static and dynamic event-triggered sliding mode controllers concerning the average consensus issues for single- and double-integrator multi-agent systems with perturbations. To guarantee the consensus convergence of disturbed first- and second-order multi-agent systems, two distributed sliding manifolds with respect to an odd function are firstly devised in this paper. Secondly, two types of event-triggered mechanisms, i.e., a static event-triggering mechanism and a dynamic event-triggering mechanism, are established to improve the utilization efficiency of network resources and avoid the continuous communication with neighbors. In both event-triggered sliding mode control strategies, the fully distributed event-triggered sliding mode control laws without global information of the multi-agent networks are proposed, and they can ensure the state trajectories of disturbed first- and second-order multi-agent systems to reach the average consensus. Meanwhile, the finite-time reachability of the specified sliding manifold can be guaranteed and Zeno behavior can be also averted. Thirdly, taking advantage of the Lyapunov stability theory and sliding mode control, sufficient conditions for the average consensus of single- and double-integrator continuous-time multi-agent systems are established. At the end, in order to show the validity of the proposed event-triggered sliding mode control strategies, a numerical simulation and comparative study are offered.**

***Index Terms*—Average consensus; disturbed multi-agent systems; dynamic event-triggering mechanism; fully distributed event-triggered robust controller; sliding mode control.**

I. INTRODUCTION

OVer the past decades, with the development of computer science and network communication, the cooperative control of multi-agent systems (MASs) has received growing attentions from the academia [1]–[5], and has been widely employed in practical fields such as robot formation, sensor networks, power systems and so on. Notice that the consensus control, which aims to ensure that all agents reach an agreement, is one of the basic problems in the research of cooperative control, and has attracted more and more attention. For example, the distributed consensus issues for linear and nonlinear MASs are studied via designing distributed adaptive protocols in [3]. The authors in [4] discussed the distributed consensus control problem for a class of disturbed second-order MASs with leader and leaderless cases. Therefore, the consensus control of disturbed MASs governed by single- and double-integrator dynamics is studied in this paper.

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D. Yao and H. Li are with the School of Automation and the Guangdong Province Key Laboratory of Intelligent Decision and Cooperative Control, Guangdong University of Technology, Guangzhou 510006, China (email: deyinyao2013@gmail.com, lihongyi2009@gmail.com).

Y. Shi is with the Department of Mechanical Engineering, University of Victoria, Victoria, BC V8W 3P6, Canada (email: yshi@uvic.ca).

It merits noticing that the aforementioned works on the consensus control are in a manner of time-triggered control which may bring a heavy communication burden. To cope with the communication burden problem for MASs, the event-triggered control strategy has received increasing attention in the control community. The fundamental idea of event-triggered control method is to replace periodic control with aperiodic control to cut down the frequency of data transmission, which has gained an extremely considerable research in [6]–[13] and the references therein. For example, in order to better regulate the triggering intervals, the authors in [11] studied the average consensus issue for first-order MASs by designing dynamic event-triggered control protocols under undigraphs. To avert the demand for a priori knowledge of the minimal positive eigenvalue of the Laplacian matrix, the authors developed an event-based adaptive control strategy to address the robust cooperative output regulation issue for linear MASs with disturbances and uncertainties in [12]. It is worth stressing that, in the aforementioned works, some controller parameters and the triggering transmission mechanism are determined by the global smallest positive eigenvalue of the Laplacian matrix of the communication topology, i.e., the designed event-triggered control laws were not fully distributed in fact. In order not to require any global information of multi-agent network, the fully distributed adaptive controllers for linear MASs are devised in [14] to obtain the static triggering condition. But, the fully dynamic event-triggered control protocol that can strike a balance between decreasing the frequency of data transmission and guaranteeing the average consensus for disturbed continuous-time MASs determined by single- and double-integrator dynamics has not yet been considered.

On the other hand, it needs to be especially emphasized that sliding mode control (SMC) is an efficacious robust control scheme to suppress external disturbances due to its strong adaptability to perturbations and parameter uncertainties [15], [16], and there are some entertaining works about the SMC-based consensus have been extensively studied for MASs [17]–[20], where a robust sliding mode controller is designed for heterogeneous higher-order nonlinear MASs suffering from mismatched uncertainties in [17]. Under these SMC methods, it is of great importance to handle the event-triggered communication issue for dynamic systems, such as [21]–[23]. Of them, to eliminate the claim for continuous state measurement, the periodic event-triggered mechanism of the SMC design issue for linear time-invariant systems is investigated in [22]. However, there are few studies on the fully distributed static and dynamic event-triggered SMC protocols for disturbed continuous-time MASs with single- and double-integrator dynamics.

Enlightened by what has been discussed aforementioned, we propose the fully distributed static and dynamic event-triggered SMC protocols for disturbed single- and double-integrator continuous-time MASs to cope with the problems of average consensus and the utilization of limited communication resources simultaneously in this paper, respectively. The core contributions are summarized as follows.

i) This article proposes the distributed static and dynamic triggering control schemes for disturbed single- and double-integrator continuous-time MASs to reduce the continuous communication between agents equipped with many kinds of sensors and actuators. Unlike [11], [24] without considering external disturbances, the dynamic triggering rule proposed in this work introduces an extra positive signal to lessen the amount of transmitted data, and analyzes the consensus with considering disturbances. Besides, Zeno phenomena in both event-triggered control strategies are ruled out by proving that the activated time sequence is divergent for each agent.

ii) A more common framework for the SMC-based average consensus performance of single- and double-integrator continuous-time MASs subjected to disturbances is established. Under such a framework, the distributed discontinuous sliding surface with an odd function is devised which is different from [25]. Distinguished from [26], [27], the distributed discontinuous sliding manifold with decoupled agent states is constructed so that the design of static and dynamic sliding mode controllers is fully distributed without any knowledge of global network parameters.

iii) The average consensus control issues for disturbed first- and second-order continuous-time MASs are, respectively, studied by utilizing the fully distributed static and dynamic event-triggered SMC protocols. Meanwhile, in comparison with the previous results [23], [28], the designed control law does not need to involve the extra dynamic variable value of the dynamic event-triggering mechanism in our work, which makes the control implementation easier and more convenient. Via the Lyapunov stability theory and SMC, sufficient conditions for the average consensus of single- and double-integrator MASs are derived.

Notations: The transpose of a matrix $\mathscr{A}$ is denoted by $\mathscr{A}^{\mathrm{T}}$. $\mathbb{R}^n$ shows an $n \times 1$ real column vector. Define a vector $X = [X_1, X_2, \dots, X_N]^{\mathrm{T}} \in \mathbb{R}^N$, whose 1-norm is denoted as $|X| = \sum_{i=1}^{N} |X_i|$.

II. Preliminaries and problem description

A. Graph theory

In this paper, among the agents, we utilize an undirected graph $\mathscr{S} = (\mathscr{W}, \mathscr{D}, P)$ to characterize the information interaction with $\mathscr{W} = \{1, 2, \dots, N\}$ and $\mathscr{D} = \{(i, j), i, j \in \mathscr{W}\}$ being the node set and the edge set. In addition, the weighted adjacency matrix is shown by $P = [a_{ij}]_{N\times N}$ with $a_{ij} = 1$ if agent i and agent j are connected; otherwise, $a_{ij} = 0$. The Laplacian matrix of the undirected graph $\mathscr{S}$ is expressed by $\mathscr{R} = [l_{ij}]_{N\times N}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$, and $l_{ij} = -a_{ij}$ if $i \neq j$.

III. Disturbed single-integrator continuous-time MASs

A. Static triggering mechanism

There exist N agents in this part for the disturbed single-integrator continuous-time MAS. The dynamics of the i^{th} agent are as follows:

$$\dot{x}_i(t) = u_i(t) + d_i(t), \quad i = 1, 2, \dots, N \tag{1}$$

where $x_i(t) \in \mathbb{R}$, $u_i(t) \in \mathbb{R}$ and $d_i(t) \in \mathbb{R}$ are the state, control input and disturbance, respectively.

The control objective of the paper is to establish a type of fully distributed event-based control protocols $u_i(t)$ for disturbed continuous-time MASs to guarantee that the average consensus is achieved, while reducing controller update frequency, averting continuous information communication, and excluding Zeno behavior.

To achieve the average consensus for disturbed continuous-time MASs, a lemma, a definition and some mild hypotheses are stated.

Lemma 1: [11] Consider the continuous-time MAS in (1) and the undirected graph $\mathscr{S}$. The states of all agents converge to a common point, which equals to $\frac{1}{N}\sum_{i=1}^{N} x_i(0)$ for $\forall t \geq 0$.

Definition 1: [19] $\mathrm{sig}(\cdot) : \mathbb{R}^k \to \mathbb{R}^k$ represents an odd function, whose definition is presented as follows:

$$\mathrm{sig}(\varphi)^{\gamma} = [|\varphi_1|^{\gamma}\mathrm{sign}(\varphi_1), \dots, |\varphi_k|^{\gamma}\mathrm{sign}(\varphi_k)]^{\mathrm{T}},$$

in which $\gamma > 0$, $|\varphi_i|$ stands for the absolute value of φ_i for $i = 1, 2, \dots, k$, $\varphi = [\varphi_1, \varphi_2, \dots, \varphi_k]^{\mathrm{T}}$, and $\mathrm{sign}(\cdot)$ denotes the sign function.

Assumption 1: [9], [25] The undirected graph $\mathscr{S}$ is connected.

Assumption 2: [22], [26] Assume that the disturbance $d_i(t)$ is norm-bounded, i.e., $|d_i(t)| \leq \varpi_i$ with $\varpi_i > 0$ being a known constant.

Following the SMC design principle in [16], the distributed sliding manifold is devised for the system (1) by

$$S_i(t) = x_i(t) + \phi_i(t), \tag{2}$$

where $\dot{\phi}_i(t) = m_1 \sum_{j=1}^{N} a_{ij}\mathrm{sig}(x_i(t) - x_j(t))^{c_1}$ with $m_1 > 0$ and odd integer $c_1 > 1$ being designed parameters.

Now, we describe how the SMC method is employed under the event-triggered control mechanism. Define a relative state variable as $\tilde{x}_i(t) = -m_1 \sum_{j=1}^{N} a_{ij}\mathrm{sig}(x_i(t) - x_j(t))^{c_1}$ that can be available by the i^{th} agent. The fully distributed event-triggered control protocol is constructed by

$$u_i(t) = \tilde{x}_i(t_k^i) - K_{1i}\mathrm{sign}(S_i(t_k^i)), \quad t \in [t_k^i, t_{k+1}^i) \tag{3}$$

$$S_i(t_k^i) = x_i(t_k^i) - \int_{t_k^i}^{t} \tilde{x}_i(t_k^i)dt, \quad t \in [t_k^i, t_{k+1}^i) \tag{4}$$

where $K_{1i} > 0$, and t_k^i are the lastest activated instants of the i^{th} agent determined by the triggering rule constructed in (5). That is, when the triggering condition in (5) is met, $\tilde{x}_i\left(t_k^i\right)$ and $x_i\left(t_k^i\right)$ are broadcasted to calculate $S_i(t_k^i)$ and update $u_i(t)$.

Remark 1: It should be noticed that, referring to [29], the Filippov solutions for the closed-loop system in (1) with the control law in (3) exist.

Remark 2: The fully distributed event-triggered protocol in (3) of this work includes a sig function and a signum function, so it is switched. Hence, to restrain the unlimited switching of controllers, the tanh function replaces the signum function to facilitate the derivation of the sign function, and the approximation function $\text{sign}(S_i(t)) \approx \frac{S_i(t)}{|S_i(t)|+\hbar}$ [27] with a small constant $\hbar > 0$ is simulated to take the place of the signum function in Section V.

To verify the finite-time reachability of the assigned switching surface, an error variable is established in the following form:

$$e_x^i(t) = \tilde{x}_i(t) - K_{1i}\text{sign}(S_i(t)) - \tilde{x}_i(t_k^i) + K_{1i}\text{sign}(S_i(t_k^i)),$$

Below, we construct a distributed fixed threshold triggering rule as

$$t_{k+1}^i = \inf\{t > t_k^i \,|\, |e_x^i(t)| \geq \pi_i\}, \quad \pi_i > 0 \tag{5}$$

for each agent i, where the requirement $\pi_i + \varpi_i + \varsigma_i \leq K_{1i}$ holds with ς_i being a positive scalar.

The finite-time reachability of the sliding mode surface can be ensured by Theorem 1, the system state can thus begin the sliding motion after reaching the switching surface in (2).

Theorem 1: Under the fully distributed robust control law in (3) and the fixed threshold triggering rule in (5), the states of the disturbed single-integrator continuous-time MAS in (1) can be driven to the prescribed sliding manifold in (4) within a limited time $t_i^\star \leq \frac{\sqrt{\sum_{i=1}^N S_i^2(0)}}{\varsigma}$.

Proof: The Lyapunov function is selected as

$$V_1(t) = \frac{1}{2}\sum_{i=1}^N S_i^2(t). \tag{6}$$

Taking the time-derivative of (6) results in

$$\dot{V}_1(t) = \sum_{i=1}^N S_i(t)\left(u_i(t) + d_i(t) - \tilde{x}_i(t)\right), \tag{7}$$

then, by employing the error variable $e_x^i(t)$, Equation (3), the triggering rule in (5) and Assumption 2, one further obtains

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^N S_i(t)\left(-e_x^i(t) - K_{1i}\text{sign}(S_i(t)) + d_i(t)\right)\\ &\leq -\sum_{i=1}^N |S_i(t)|\left(K_{1i} - \pi_i - \varpi_i\right).\end{aligned} \tag{8}$$

According to the design constraint $\pi_i + \varpi_i + \varsigma_i \leq K_{1i}$, Equation (8) is further derived as $\dot{V}_1(t) \leq -\sum_{i=1}^N \varsigma|S_i(t)|$, where $\varsigma = \min_i\{\varsigma_i\} > 0$. Hence, the reaching time for each agent is reckoned as $t_i^\star \leq \frac{\sqrt{2V_1(0)}}{\varsigma}$.

Consequently, although there has restricted network bandwidth, the states of the closed-loop MAS in (1) governed by single-integrator dynamics can still be driven into the switching surface within a limited period of time by constituting a fully distributed controller in (3). The proof is finished. ■

B. Stability analysis of the closed-loop system

After the system state reaches the sliding mode of the i^{th} agent, we can get $S_i = 0$ and $\dot{S}_i = 0$, leading to

$$\dot{x}_i(t) = \tilde{x}_i(t) = -m_1\sum_{j=1}^N a_{ij}\text{sig}\left(x_i(t) - x_j(t)\right)^{c_1}.$$

Based on Assumption 1 and Definition 1, we can get

$$\begin{aligned}-\frac{1}{N}\sum_{j=1}^N \dot{x}_j(t) &= \frac{m_1}{N}\sum_{i=1}^N\sum_{j=1}^N a_{ij}\text{sig}\left(x_i(t) - x_j(t)\right)^{c_1}\\ &= 0.\end{aligned} \tag{9}$$

Based on Equation (9), the average consensus $e_{xi}^1(t) = x_i(t) - \frac{1}{N}\sum_{j=1}^N x_j(t)$ is defined to calculate the sliding mode dynamics as

$$\dot{e}_{xi}^1(t) = -m_1\sum_{j=1}^N a_{ij}\text{sig}\left(e_{xi}^1(t) - e_{xj}^1(t)\right)^{c_1}. \tag{10}$$

Theorem 2: When the states of the MAS in (1) reach the pre-designed switching surface in (4) in finite time, the states of the sliding mode dynamics in (10) will reach the average consensus under the fully distributed event-triggered SMC law in (3), i.e., the closed-loop continuous-time MAS in (1) can achieve average consensus.

Proof: The candidate Lyapunov function is constructed as

$$V_2(t) = \frac{1}{2}\sum_{i=1}^N (e_{xi}^1(t))^2. \tag{11}$$

The time-derivative of $V_2(t)$ in (11) is calculated as

$$\begin{aligned}\dot{V}_2(t) &= \sum_{i=1}^N e_{xi}^1(t)\,\dot{e}_{xi}^1(t)\\ &= -\frac{m_1}{2}\sum_{i=1}^N\sum_{j=1}^N a_{ij}(e_{xi}^1(t) - e_{xj}^1(t))\\ &\quad \times \text{sig}\left(e_{xi}^1(t) - e_{xj}^1(t)\right)^{c_1}.\end{aligned}$$

According to Definition 1, one has

$$\begin{aligned}&(e_{xi}^1(t) - e_{xj}^1(t))\text{sig}\left(e_{xi}^1(t) - e_{xj}^1(t)\right)^{c_1}\\ &= (e_{xi}^1(t) - e_{xj}^1(t))|e_{xi}^1(t) - e_{xj}^1(t)|^{c_1}\\ &\quad \times \text{sign}\left(e_{xi}^1(t) - e_{xj}^1(t)\right)\\ &= |e_{xi}^1(t) - e_{xj}^1(t)|^{1+c_1},\end{aligned}$$

which leads to $\dot{V}_2(t) < 0$. As a result, according to Lemma 1, the disturbed continuous-time MAS in (1) can achieve average consensus performance. The proof is completed. ■

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C. Feasibility of static triggering function

The activated instants are represented by a monotonically increasing sequence that is denoted as $t_0^i, t_1^i, \ldots, t_k^i, \ldots$ with $t_0^i = 0$, where $t_{k+1}^i - t_k^i = \theta_k^i$ for positive integer k. Therefore, between any two consecutively activated instants, the lower bound on the inter-event interval θ_k^i will be provided in the following.

Theorem 3: By applying the fully distributed robust control signal in (3) and the static triggering function in (5) to the system in (1), there exists a positive lower bound for the inter-event interval θ_k^i such that Zeno behavior does not occur.

Proof: From the triggering condition in (5) for $t \in [t_k^i, t_{k+1}^i)$, according to $\tilde{x}_i(t) = -m_1 \sum_{j=1}^N a_{ij} \mathrm{sig}(x_i(t) - x_j(t))^{c_1}$, we can obtain

$$\frac{d(|e_x^i(t)|)}{dt} \le c_1 m_1 \sum_{j=1}^N a_{ij} |\left(x_i(t) - x_j(t)\right)^{c_1 - 1}| \times \sum_{j=1}^N a_{ij} |u_i(t) + d_i(t) - u_j(t) - d_j(t)| + K_{1i} |1 - \tanh^2(\sigma S_i(t))| |\sigma \dot{S}_i(t)| \quad (12)$$

with $\sigma >> 1$ being a constant.

From Assumption 2 and [26, Theorem 3], the inequality in (12) is calculated as follows:

$$\frac{d(|e_x^i(t)|)}{dt} \le c_1 m_1 \sum_{j=1}^N a_{ij} |\left(x_i(t) - x_j(t)\right)^{c_1 - 1}| \Big(N\varpi_i + \sum_{j=1}^N \varpi_j + \sum_{j=1}^N a_{ij} |u_i(t) - u_j(t)| \Big) + K_{1i}\sigma \Big(|u_i(t)| + \varpi_i + |m_1 \sum_{j=1}^N a_{ij} (x_i(t) - x_j(t))^{c_1}| \Big). \quad (13)$$

It is well known that the event will not be triggered until $|e_x^i(t)| = \pi_i$ according to the criterion (5). Further considering (13), we have

$$\pi_i \le |e_x^i(t)| \le \theta_k^i c_1 m_1 \sum_{j=1}^N a_{ij} |\left(x_i(t) - x_j(t)\right)^{c_1 - 1}| \times (\sum_{j=1}^N a_{ij} |u_i(t) - u_j(t)| + N\varpi_i + \sum_{j=1}^N \varpi_j) + \theta_k^i K_{1i}\sigma \times (|u_i(t)| + \varpi_i + |m_1 \sum_{j=1}^N a_{ij} \left(x_i(t) - x_j(t)\right)^{c_1}|). \quad (14)$$

Therefore, the inter-event interval θ_k^i is computed by $\theta_k^i \ge \frac{\pi_i}{\varepsilon_i} > 0$ with

$$\varepsilon_i = c_1 m_1 \sum_{j=1}^N a_{ij} |\left(x_i(t) - x_j(t)\right)^{c_1 - 1}| \times (\sum_{j=1}^N a_{ij} |u_i(t) - u_j(t)| + N\varpi_i + \sum_{j=1}^N \varpi_j) + K_{1i}\sigma \times (|u_i(t)| + \varpi_i + |m_1 \sum_{j=1}^N a_{ij} \left(x_i(t) - x_j(t)\right)^{c_1}|),$$

which shows that the Zeno behavior can be excluded. This completes the proof. ■

D. Dynamic triggering mechanism

Based on Equation (5), a novel distributed dynamic event-triggering condition for the system (1) is devised with the following form:

$$\Pi_i(t) = |e_x^i(t)| - \frac{1}{\iota_i}\pi_i - \zeta_i(t), \quad (15)$$

$$\dot{\zeta}_i(t) = -h_i \zeta_i(t) + (\pi_i - \iota_i |e_x^i(t)| + \iota_i \zeta_i(t)) |S_i(t)|, \quad (16)$$

where $\pi_i > 0$, $\iota_i > 1$, $h_i > \iota_i |S_i(t)|$ and $\zeta_i(0) > 0$.

Theorem 4: The state trajectories of the system in (1) are driven into the switching surface devised in (4) within a finite time by the fully distributed event-triggering controller in (3) and the dynamic event-triggered communication rule in (15).

Proof: Following the dynamic triggering rule in (15), the system state is not triggered as $t \in [t_k^i, t_{k+1}^i)$. Thus, we have

$$\pi_i - \iota_i |e_x^i(t)| \ge -\iota_i \zeta_i(t), \quad t \in \left[t_k^i, t_{k+1}^i\right).$$

In such a scenario, one has $\dot{\zeta}_i(t) \ge -h_i \zeta_i(t)$, thus implying $\zeta_i(t) \ge \exp(-h_i t)\zeta_i(0) > 0$.

Therefore, we construct the Lyapunov function as follows:

$$V_3(t) = \frac{1}{2} \sum_{i=1}^N S_i^2(t) + \sum_{i=1}^N \zeta_i(t). \quad (17)$$

Via (3), (15) and (16), the time-derivative of $V_3(t)$ is $\dot{V}_3(t) \le -\sum_{i=1}^N |S_i(t)| (K_{1i} - \pi_i - \varpi_i)$. As a result, we can select some suitable scalars K_{1i}, ϖ_i and π_i to make the constraint requirement $\pi_i + \varpi_i + \omega_i \le K_{1i}$ hold, so $\dot{V}_3(t) \le -\omega \sum_{i=1}^N |S_i(t)|$ establishes with $\omega = \min_i \{\omega_i\} > 0$. Afterwards, the states of (1) can reach the switching surface in finite time. The proof is completed. ■

Theorem 5: Under the fully distributed robust controller in (3), the dynamic event-triggering rule in (15) and the internal dynamic variable in (16), the inter-event interval $\bar{\theta}_k^i$ has a positive lower bound to eliminate Zeno behavior in the closed-loop system.

Proof: In light of the dynamic event-triggering rule in (15) for $t \in [t_k^i, t_{k+1}^i)$, we can obtain

$$\frac{d\left(|e_x^i(t)|\right)}{dt} \le c_1 m_1 \sum_{j=1}^N a_{ij} |\left(x_i(t) - x_j(t)\right)^{c_1 - 1}| \times \sum_{j=1}^N a_{ij} |u_i(t) + d_i(t) - u_j(t) - d_j(t)| + K_{1i} |1 - \tanh^2(\bar{\sigma} S_i(t))| |\bar{\sigma} \dot{S}_i(t)| \quad (18)$$

with $\bar{\sigma} >> 1$ representing a constant.

According to Theorem 4 and $\zeta_i(t) \ge \exp(-h_i t)\zeta_i(0) > 0$, for each agent, for $\forall t \in [t_k^i, t_{k+1}^i)$, defining $t_{k+1}^i - t_k^i = \bar{\theta}_k^i$ for positive integer k, we can achieve the inter-event interval $\bar{\theta}_k^i \ge \frac{\frac{1}{\iota_i}\pi_i + \zeta_i(0)\exp(-h_i t_{k+1}^i)}{\psi_i}$ with the variable ψ_i being defined as $\psi_i = c_1 m_1 \sum_{j=1}^N a_{ij} |(x_i(t) -$

$x_j(t))^{c_1-1}|(\sum_{j=1}^{N} a_{ij}|u_i(t)-u_j(t)|+N\varpi_i+\sum_{j=1}^{N}\varpi_j)+K_{1i}\bar{\sigma}(|u_i(t)|+\varpi_i+|m_1\sum_{j=1}^{N}a_{ij}(x_i(t)-x_j(t))^{c_1}|)$, Zeno behavior is thus excluded. The proof is completed. ■

Remark 3: It should be especially pointed out that the time-derivative of the Lyapunov function $V_3(t)$ is negative in line with Theorem 4, so the time-derivative of the internal dynamic variable $\zeta_i(t)$ is also negative. Therefore, the trajectory of the variable $\zeta_i(t)$ is asymptotically stable.

IV. Disturbed Double-Integrator Continuous-Time MASs

A. Static event-triggered control law design

This section studies the MAS consisting of agents with double-integrator dynamics modeled by

$$\begin{cases} \dot{x}_i(t)=\upsilon_i(t), \\ \dot{\upsilon}_i(t)=d_i(x_i(t),\upsilon_i(t),t)+u_i(t), \end{cases} \tag{19}$$

for $i=1,2,\ldots,N$, where the position, velocity, control input and external disturbance are independently expressed by $x_i(t),\upsilon_i(t)\in\mathbb{R}$, $u_i(t)\in\mathbb{R}$ and $d_i(x_i(t),\upsilon_i(t),t)\in\mathbb{R}$.

Then, the distributed sliding mode surface is designed by

$$S_i(t)=\dot{x}_i(t)+\xi_i(t) \tag{20}$$

with

$$\begin{aligned}\dot{\xi}_i(t)=&\ \alpha_1\sum_{j=1}^{N}a_{ij}\mathrm{sig}\left(x_i(t)-x_j(t)\right)^{\beta_1}\\ &+\alpha_2\sum_{j=1}^{N}a_{ij}\mathrm{sig}\left(\upsilon_i(t)-\upsilon_j(t)\right)^{\beta_2},\end{aligned} \tag{21}$$

in which the designed parameters $\alpha_1>0$, $\alpha_2>0$, and odd integers $\beta_1>1$ and $\beta_2>1$.

Based on (21), we define a relative measurable state variable as $\tilde{z}_i(t)=-\alpha_1\sum_{j=1}^{N}a_{ij}\mathrm{sig}(x_i(t)-x_j(t))^{\beta_1}-\alpha_2\sum_{j=1}^{N}a_{ij}\mathrm{sig}(\upsilon_i(t)-\upsilon_j(t))^{\beta_2}$ for agent i. For $t\in[t_k^i,t_{k+1}^i)$, the fully distributed event-triggered SMC protocol and sliding manifold are then devised by

$$u_i(t)=\tilde{z}_i\left(t_k^i\right)-K_{2i}\mathrm{sign}(S_i(t_k^i)), \tag{22}$$

$$S_i(t_k^i)=\upsilon_i(t_k^i)-\int_{t_k^i}^{t}\tilde{z}_i\left(t_k^i\right)dt \tag{23}$$

with $K_{2i}>0$ being a designed constant for each agent i. As the triggering condition in (24) is satisfied, $\tilde{z}_i\left(t_k^i\right)$ and $\upsilon_i\left(t_k^i\right)$ are propagated to update the sliding mode surface $S_i(t_k^i)$ and the control law $u_i(t)$ in (22).

In order to achieve the average consensus of the disturbed double-integrator continuous-time MAS in (19), we define the following error variable:

$$e^i(t)=\tilde{z}_i(t)-K_{2i}\mathrm{sign}(S_i(t))-\tilde{z}_i\left(t_k^i\right)+K_{2i}\mathrm{sign}(S_i(t_k^i)).$$

According to the definition of the error variable $e^i(t)$, a distributed static trigger mechanism is built with the following form:

$$t_{k+1}^i=\inf\{t>t_k^i||e^i(t)|\geq\varrho_i\} \tag{24}$$

with $\varrho_i>0$ representing the trigger threshold for each agent. For constant $\rho_i>0$, it satisfies the condition $\rho_i+\varpi_i+\varrho_i\leq K_{2i}$.

Theorem 6: The trajectories of the closed-loop double-integrator continuous-time MAS in (19) are driven to the preconstructed sliding mode surface in (23) in finite time based on the fully distributed robust control protocol in (22) and the static triggering rule in (24).

Proof: Construct the Lyapunov function as

$$V_4(t)=\frac{1}{2}\sum_{i=1}^{N}S_i^2(t). \tag{25}$$

Here, on account of Assumption 2, the robust controller in (22) and the triggering mechanism in (24), the time-derivative of the Lyapunov function $V_4(t)$ is then derived as

$$\begin{aligned}\dot{V}_4(t)=&\sum_{i=1}^{N}S_i(t)\left(e^i(t)-K_{2i}\mathrm{sign}(S_i(t))+d_i(t)\right)\\ \leq&\sum_{i=1}^{N}|S_i(t)|\left(\left|e^i(t)\right|-K_{2i}+\varpi_i\right)\\ \leq&-\sum_{i=1}^{N}|S_i(t)|\left(K_{2i}-\varrho_i-\varpi_i\right),\end{aligned} \tag{26}$$

by utilizing the condition $\rho_i+\varpi_i+\varrho_i\leq K_{2i}$, the inequality in (26) is then converted into $\dot{V}_4(t)\leq-\sum_{i=1}^{N}\rho|S_i(t)|$ with $\rho=\min_i\{\rho_i\}>0$. In this situation, the position and velocity trajectories of the double-integrator MAS in (19) can be forced to the sliding mode surface within a limited period of time, which is calculated as $t_i^\star\leq\frac{\sqrt{2V_4(0)}}{\rho}$. The proof is thus completed. ■

B. Stability analysis of sliding mode dynamics

When the designed distributed control law forces the system state to move on the sliding manifold in (23), one has

$$\begin{cases} \dot{x}_i(t)=\upsilon_i(t), \\ \dot{\upsilon}_i(t)=-\alpha_1\sum_{j=1}^{N}a_{ij}\mathrm{sig}\left(x_i(t)-x_j(t)\right)^{\beta_1} \\ \qquad\quad -\alpha_2\sum_{j=1}^{N}a_{ij}\mathrm{sig}\left(\upsilon_i(t)-\upsilon_j(t)\right)^{\beta_2}. \end{cases}$$

Since the multi-agent network under consideration is undirected, β_1 and β_2 are odd integers, and $\mathrm{sig}(\cdot)$ is an odd function, it is therefore obtained that $-\frac{1}{N}\sum_{j=1}^{N}\dot{\upsilon}_j(t)=0$. Let $e_{xi}(t)=x_i(t)-\frac{1}{N}\sum_{j=1}^{N}x_j(t)$ and $e_{\upsilon i}(t)=\upsilon_i(t)-\frac{1}{N}\sum_{j=1}^{N}\upsilon_j(t)$, and then we have

$$\begin{cases} \dot{e}_{xi}(t)=e_{\upsilon i}(t), \\ \dot{e}_{\upsilon i}(t)=-\alpha_1\sum_{j=1}^{N}a_{ij}\mathrm{sig}\left(e_{xi}(t)-e_{xj}(t)\right)^{\beta_1} \\ \qquad\quad -\alpha_2\sum_{j=1}^{N}a_{ij}\mathrm{sig}\left(e_{\upsilon i}(t)-e_{\upsilon j}(t)\right)^{\beta_2}, \end{cases} \tag{27}$$

which stands for the sliding mode dynamics of the closed-loop system in (19).

Based on the Lyapunov stability theory, the stability analysis of the sliding mode dynamics is given in the following.

Theorem 7: The states of the resulting dynamic system in (27) will achieve the average consensus utilizing the fully

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distributed event-triggered control law in (22) when reaching the decoupled sliding manifold in (23).

Proof: Based on Lemma 1, for the dynamic system in (27), we select the following candidate Lyapunov functional

$$V_5(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\int_0^{e_{xi}(t)-e_{xj}(t)} \alpha_1 a_{ij}\mathrm{sig}(\vartheta)^{\beta_1}\, d\vartheta + \frac{1}{2}\sum_{i=1}^{N} e_{vi}^2(t). \quad (28)$$

Differentiating the Lyapunov function in (28) with respect to time t and employing the equation (27) give rise to

$$\begin{aligned}\dot{V}_5(t) &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_1 a_{ij}\mathrm{sig}(e_{xi}(t)-e_{xj}(t))^{\beta_1} \\ &\quad \times (e_{vi}(t)-e_{vj}(t)) + \sum_{i=1}^{N} e_{vi}(t)\dot{e}_{vi}(t) \\ &= -\frac{\alpha_2}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(e_{vi}(t)-e_{vj}(t)) \\ &\quad \times \mathrm{sig}(e_{vi}(t)-e_{vj}(t))^{\beta_2}.\end{aligned}$$

Because the following equation holds

$$\begin{aligned}&(e_{vi}(t)-e_{vj}(t))\mathrm{sig}(e_{vi}(t)-e_{vj}(t))^{\beta_2} \\ &= |e_{vi}(t)-e_{vj}(t)|^{1+\beta_2},\end{aligned}$$

$\dot{V}_5(t) < 0$ is established. Therefore, the states of disturbed double-integrator continuous-time MASs achieve average consensus. The proof is completed. ■

C. Reasonability of static triggering condition

In the same way, a monotonically increasing time sequence $0 = t_0^i < t_1^i < \ldots < t_k^i < t_{k+1}^i < \ldots$ denotes the activated instants, where $t_{k+1}^i - t_k^i = \chi_k^i$. Then, the difference χ_k^i between successive activated instants t_k^i and t_{k+1}^i has an infimum time calculated in the sequel.

Theorem 8: The state of the closed-loop system with the fully distributed robust controller in (22) and the static event-triggering communication rule in (24) does not produce Zeno behavior.

Proof: Similar to the proof procedure of Theorem 3 in Section III-A, by $\tilde{z}_i(t) = -\alpha_1\sum_{j=1}^{N} a_{ij}\mathrm{sig}(x_i(t)-x_j(t))^{\beta_1} - \alpha_2\sum_{j=1}^{N} a_{ij}\mathrm{sig}(v_i(t)-v_j(t))^{\beta_2}$, the inter-event interval χ_k^i between any two consecutive activated moments is computed by $\chi_k^i \geq \frac{\varrho_i}{\kappa_i} > 0$ with

$$\begin{aligned}\kappa_i &= \alpha_1\beta_1\sum_{j=1}^{N} a_{ij}|(x_i(t)-x_j(t))^{\beta_1-1}|\sum_{j=1}^{N} a_{ij}|v_i(t)-v_j(t)| \\ &\quad + \alpha_2\beta_2\sum_{j=1}^{N} a_{ij}|(v_i(t)-v_j(t))^{\beta_2-1}| \\ &\quad \times \Big(\sum_{j=1}^{N} a_{ij}|u_i(t)-u_j(t)| + N\varpi_i + \sum_{j=1}^{N}\varpi_j\Big) \\ &\quad + K_{2i}\mu\Big(\alpha_1|\sum_{j=1}^{N} a_{ij}(x_i(t)-x_j(t))^{\beta_1}| \\ &\quad + |u_i(t)| + \varpi_i + \alpha_2|\sum_{j=1}^{N} a_{ij}(v_i(t)-v_j(t))^{\beta_2}|\Big),\end{aligned}$$

which can rule out Zeno phenomenon. ■

D. Dynamic event-triggered control law design

In order to enhance the efficiency of the event triggering controller, based on the static one in (24), a novel distributed dynamic event-triggering rule for disturbed second-order MASs is designed as follows:

$$\Omega_i(t) = |e^i(t)| - \epsilon_i\varrho_i - \eta_i(t), \quad (29)$$

$$\dot{\eta}_i(t) = -l_i\eta_i(t) + (\varrho_i + \frac{1}{\epsilon_i}\eta_i(t) - \frac{1}{\epsilon_i}|e^i(t)|)|S_i(t)|, \quad (30)$$

where $\varrho_i > 0$, $0 < \epsilon_i < 1$, $l_i\epsilon_i > |S_i(t)|$ and $\eta_i(0) > 0$.

Before presenting Theorem 9, we present the convergence results of the internal dynamic variable provided in the following lemma.

Lemma 2: For the dynamic event triggering condition in (29), it always holds that $\eta_i(t) \geq \exp(-l_i t)\eta_i(0) > 0$ according to [30].

Theorem 9: The states of the system in (19) will be forced to the sliding manifold in (23) within a finite time by employing the fully distributed protocol in (22), the dynamic event-triggering rule in (29) and the internal dynamic variable in (30).

Proof: Based on Lemma 2, the Lyapunov function is chosen as follows:

$$V_6(t) = \frac{1}{2}\sum_{i=1}^{N} S_i^2(t) + \sum_{i=1}^{N}\eta_i(t). \quad (31)$$

By exploiting equations (22), (29) and (30), we take the time-derivative of function $V_6(t)$ as follows:

$$\begin{aligned}\dot{V}_6(t) \leq &-\sum_{i=1}^{N}|S_i(t)|(K_{2i}-\varrho_i-\varpi_i) \\ &-\sum_{i=1}^{N}(l_i - \frac{1}{\epsilon_i}|S_i(t)|)\eta_i(t). \quad (32)\end{aligned}$$

Due to the fact that $l_i\epsilon_i > |S_i(t)|$, the inequality in (32) can be deduced as $\dot{V}_6(t) \leq -\sum_{i=1}^{N}|S_i(t)|(K_{2i}-\varrho_i-\varpi_i)$. Thus, it is feasible to find some proper parameters K_{2i}, ϱ_i and ϖ_i to guarantee that the constraint condition $\varrho_i + \varpi_i + \delta_i \leq K_{2i}$ is established, then we have $\dot{V}_6(t) \leq -\delta\sum_{i=1}^{N}|S_i(t)|$ where $\delta = \min_i\{\delta_i\} > 0$. Under such a case, the continuous-time MAS states in (19) can be driven to the sliding surface in (23) within finite time. The proof is finished. ■

Remark 4: The stability analysis of the sliding mode dynamics is omitted here because the proof can refer to Section IV-B, and the feasibility analysis of the dynamic event-triggering mechanism in (29) is also omitted.

Remark 5: In this paper, the distributed dynamic event-triggering condition constructed in (29) not only contains state variables $\tilde{z}_i(t_k^i)$ and $S_i(t_k^i)$ in (23), but also includes the dynamical change parameter $\eta_i(t)$ determined by (30). Therefore, we can select some appropriate parameters l_i, ϵ_i and ϱ_i to guarantee that the system state can reach the preestablished sliding manifold in finite time and the activated time sequence will not produce Zeno behavior.

V. Simulation Results

In order to validate the efficacy of the proposed dynamic event-triggered SMC algorithm in (29) in the paper, we provide a numerical simulation and comparative study.

The Laplacian matrix of the disturbed double-integrator continuous-time MAS in (19) under investigation in this paper including four agents is provided as

$$\mathscr{R} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

The model of the i^{th} agent is characterized by

$$\begin{cases} \dot{x}_i(t) = \upsilon_i(t), \\ \dot{\upsilon}_i(t) = d_i(t) + u_i(t), \quad i = 1,2,3,4 \end{cases}$$

where $d_i(t) = 0.1\sin(9it)\exp(-0.1t)$.

The coefficients α_1, α_2, β_1, β_2 and K_{2i} in (22) are, severally, given as $\alpha_1 = \alpha_2 = 0.01$, $\beta_1 = 3.0$, $\beta_2 = 3.0$, $K_{21} = 0.3$, $K_{22} = 0.3$, $K_{23} = 0.84$ and $K_{24} = 0.12$. The corresponding parameters in (29) are chosen as $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0.72$, $\varrho_1 = 0.15$, $\varrho_2 = 0.20$, $\varrho_3 = 1.30$, $\varrho_4 = 0.20$, $l_1 = l_2 = 4.0$, $l_3 = l_4 = 2.0$. The initial values of the internal dynamic variable $\eta_i(t)$ are set as $\eta_1(0) = 1.50$, $\eta_2(0) = 2.40$, $\eta_3(0) = 3.60$ and $\eta_4(0) = 1.0$, respectively. The initial conditions of the double-integrator continuous-time MAS in (19) are given as $x_1(0) = 0.52$, $x_2(0) = 0.40$, $x_3(0) = 0.64$, $x_4(0) = 0.64$, $\upsilon_1(0) = 0.20$, $\upsilon_2(0) = 0.22$, $\upsilon_3(0) = 0.20$, $\upsilon_4(0) = 0.20$.

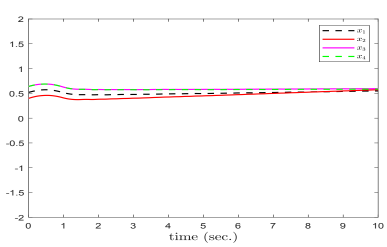

Fig. 1. Position trajectories of the double-integrator MAS under (29).

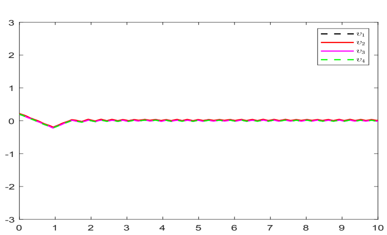

Fig. 2. Velocity trajectories of the double-integrator MAS under (29).

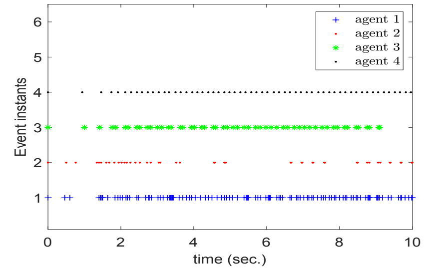

Fig. 3. The activated time instants of each agent.

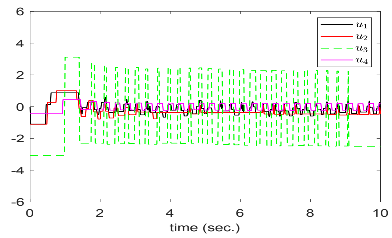

Fig. 4. Control input signals $u_i(t)$ $(i = 1,2,3,4)$.

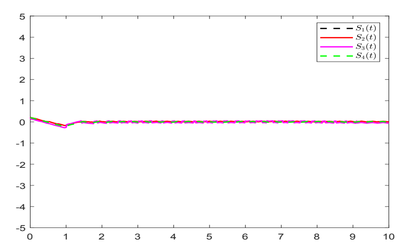

Fig. 5. Curves of sliding mode surface $S_i(t)$ $(i = 1,2,3,4)$.

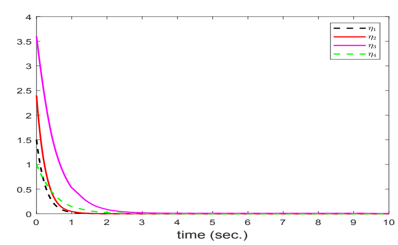

Fig. 6. Trajectories of the internal dynamic variable $\eta_i(t)$ $(i = 1,2,3,4)$.

Fig. 1 depicts the position trajectories of the disturbed double-integrator continuous-time MAS. Fig. 2 shows the velocity responses of the disturbed double-integrator continuous-time MAS. The activated communication instants of each agent are shown in Fig. 3. The time responses of the fully distributed sliding mode controllers and distributed sliding manifolds are, respectively, shown in Fig. 4 and Fig. 5. The curves of the internal dynamic variable $\eta_i(t)$ for each agent are provided in Fig. 6. According to Figs. 1–6, we can infer that the devised dynamic event-triggered sliding mode controller in (22) can stabilize the disturbed double-integrator MAS in (19) and economize on the limited communication resources.

To discuss the effect of the event-based SMC algorithm proposed in our work and the event-triggered PD control algorithm presented in [10] on the control performance for each agent, the comparative study is provided. The choice of Laplacian matrix $\mathscr{R}$ is the same as the above example. Under the same initial conditions, employing the event-triggered PD control law in [10], the position and velocity trajectories of disturbed double-integrator MASs are depicted, as shown in Fig. 7. It is inferred that the position and velocity trajectories fail to reach the average consensus in Fig. 7 due to the fluctuation of external disturbances, meaning that the control effect of the present event-based SMC algorithm in Fig. 1 and Fig. 2 is better than [10].

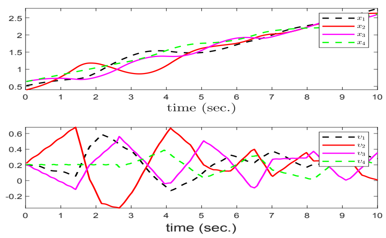

Fig. 7. Position and velocity trajectories of double-integrator MASs in [10].

VI. Conclusion

The distributed average consensus issue for disturbed single- and double-integrator MASs encountering limited network bandwidth has been investigated by proposing the fully distributed static and dynamic event-triggered SMC laws. The distributed switching surface with odd function has been devised to ensure that the states of agents can reach the specified switching surface within a limited time, and then move to the equilibrium point. The average consensus in disturbed single- and double-integrator continuous-time MASs has been achieved by integrating SMC into static/dynamic event-triggered control. The effectiveness of the proposed SMC-based event-triggered schemes for disturbed first- and second-order continuous-time MASs has been claimed by a numerical simulation and comparison experiment. Future work will focus on the application of the proposed dynamic event-triggered SMC protocol for multiple nonlinear Euler-Lagrange systems with stochastic noises and uncertainties.

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