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Brief paper

Linear quadratic optimal consensus of discrete-time multi-agent systems with optimal steady state: A distributed model predictive control approach ☆

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ABSTRACT

This paper develops a distributed model predictive control algorithm for linear quadratic optimal consensus of discrete-time multi-agent systems. The consensus state and control sequence are both optimized at every predictive step on a finite horizon and then implemented in the real system. The stability of the closed-loop system is analyzed, establishing a distributed consensus condition depending only on individual agent's local parameters. The consensus condition is then relaxed for controllable systems, making it easy to choose the weighted matrices and control period for each agent. The proposed algorithm is applied to the formation control of multi-vehicle systems verified by numerical simulations.

1. Introduction

Cooperative control of multi-agent systems, inspired by the collective behaviors in nature such as swarming of insects and flocking of birds, has drawn notable attention from researchers in different fields and played an important role in practical applications including formation flight (Ren & Beard, 2004; Wang et al., 2019), distributed filtering (Duan, Duan, Chen, & Shi, 2020; Olfati-Saber), smart grids (Wen, Yu, Liu, & Yu, 2018), and many others. Considerable work has been devoted to addressing cooperative control problems from different perspectives, such as consensus (Olfati-Saber, Fax, & Murray, 2007), formation (Ren & Beard, 2004), tracking (Lv, Wen, & Huang, 2019) and flocking (Zhan & Li, 2013), where in particular consensus is a focal topic because of its potential ability in solving many other related problems.

The critical task for solving the consensus problem is to design distributed control protocols based on local information, that is, the information of each agent and its neighbors, to achieve an agreement globally on a certain quantity of common interest. A theoretical framework is introduced in Olfati-Saber et al. (2007) for multi-agent systems, where basic concepts of consensus, design and analysis methods, and some consensus algorithms are developed. Thereafter, the control theory for solving consensus problems has gained remarkable progress for different scenarios (see Lv et al., 2019; Ren & Beard, 2005; Wen & Zheng, 2018 and the references therein). In order to get better performance of consensus, latest results are obtained for optimal consensus of multi-agent systems with different optimization objectives, such as faster convergence rate (Zhao, Liu, Wen, Ren, & Chen, 2019), less energy consumption (Sardellitti, Barbarossa, & Swami, 2012) and stronger robustness (Li & Chen, 2017).

Model predictive control (MPC), where the process model is used to forecast system dynamics for control input modification, is a powerful technique for solving the optimal consensus problem of multi-agent systems. MPC mechanism is incorporated into the classical consensus protocol of some pinning nodes so as to accelerate the convergence of consensus in Zhang, Chen, and Stan (2011). This predictive pinning control method, however, requires global information of the network, which fails to work in real time because of the huge computational and communicational burdens caused by the large scales of the networked systems. Therefore, distributed model predictive control (DMPC) has attracted increasing attention in recent years. Decomposition–coordination technique and estimator-based predictive control scheme are presented in Fawal, Georges, and Bornard (0000)

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and Gomez, Rodellar, Vea, Mantecon, and Cardona (0000), respectively, to decompose a large-scale centralized MPC problem to several small-scale local optimization problems for decentralized computation. Then, DMPC problems of networked systems are solved in Camponogara (2000) and Dong and Krogh (0000) for situations where information is allowed to be exchanged at any time and only during each control interval, respectively. On the basis of these results, some representative design methods and stability analysis for DMPC are summarized in Camponogara, Jia, Krogh, and Talukdar (2002), focusing on different information interaction mechanisms.

Recently, DMPC has been applied in consensus control of multi-agent systems due to its great efficiency and scalability. In Ferrari-Trecate, Galbusera, Marciandi, and Scattolini (2009), DMPC schemes are proposed for consensus of single- and double-integrator multi-agent systems with time-varying communication networks and constraints on agents' inputs. The results are then extended to derive analytical solutions and stability conditions for the DMPC flocking problem of double-integrator multi-agent systems in Zhan and Li (2013), based only on position measurements. For a group of agents with second-order nonlinear dynamics, a DMPC algorithm is proposed to track a desired dynamic reference in Gao, Dai, Xia, and Liu (2017), where the recursive feasibility and closed-loop stability are guaranteed by a proper design of terminal ingredients and compatibility constraints. With the application of alternating direction method of multipliers, a DMPC framework is developed in Summers and Lygeros (0000) to achieve consensus of general linear multi-agent systems, using local copies of states and inputs over a finite horizon of neighbors, which requires the local copies to be same across all coupled agents. For linear multi-agent systems with uncertainties including disturbances, estimation errors and measurement noise, in Dai, Xia, Gao, Kouvaritakis, and Cannon (2015) a cooperative stochastic DMPC algorithm is proposed to guarantee the stability of the overall system for any structure of cooperation of agents. Although the existing work provides interesting results on DMPC consensus, there is still much more to be explored for improving the consensus performance by optimizing the final consensus state as well as the control input, which greatly motivates the present investigation.

In this paper, a DMPC approach is proposed for the linear quadratic optimal consensus problem of discrete-time multi-agent systems. With the introduction of a consensus manifold, the final consensus state and input sequence are both regarded as decision variables of the optimization problem at every predictive step. In this approach, not only transient but also steady-state responses are optimized, which leads to better consensus performance in general. This is the main contribution of the paper. Then, the finite-time optimal solution solved by the method developed in Wang, Duan, Wang and Chen (2019) at the predictive step is extended to an infinite horizon through the MPC approach, where the closed-loop stability is ensured by some constraints on the weighted matrices of the agents, which constitutes another contribution of this paper. Finally, the stability condition is relaxed for controllable dynamics and a simplified condition is derived in a decentralized setting, where the individual agent's weighted matrix and control period are determined independently, thereby improving the scalability of the algorithm.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the sets of n-dimensional real column vectors and $m \times n$ real matrices, respectively; $\otimes$ denotes the Kronecker product of matrices; $\|\cdot\|$ denotes the Euclidean norm of vectors. $\mathrm{col}\{x_1,\ldots,x_m\} \triangleq [x_1^T,\ldots,x_m^T]^T$ denotes the collection of vectors $x_i \in \mathbb{R}^{n_i},\ i=1,\ldots,m$ and $\mathrm{diag}\{A_1,\ldots,A_m\}$ denotes the block diagonal matrix consisting of $A_i \in \mathbb{R}^{m_i\times n_i},\ i=1,\ldots,m$; for matrices $A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{m\times m}$, $\|A\|_B^2 \triangleq A^TBA$.

2. Preliminaries and problem formulation

Consider a discrete-time multi-agent system consisting of N linear time invariant (LTI) dynamical agents:

$$x_i(\tau+1) = Ax_i(\tau) + Bu_i(\tau),\ i \in \{1,2,\ldots,N\}, \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state of the ith agent, $u_i \in \mathbb{R}^m$ is its control input, and $A \in \mathbb{R}^{n\times n},\ B \in \mathbb{R}^{n\times m}$ are constant matrices.

The agents are assumed to exchange information through a communication network described by an undirected and connected graph $\mathcal{G}$. The adjacency matrix and Laplacian matrix of $\mathcal{G}$ are denoted by $\mathcal{A} = (a_{ij}) \in \mathbf{R}^{N\times N}$ and $\mathcal{L} = (l_{ij}) \in \mathbf{R}^{N\times N}$.

Lemma 1 (*Diestel, 1997*)**.** *If the graph $\mathcal{G}$ is connected, then its Laplacian matrix $\mathcal{L}$ has a simple eigenvalue 0 with associated eigenvector* 1*, and the other $n-1$ eigenvalues are positive.*

For the N agents, define the consensus manifold as

$$\mathcal{Z} = \left\{Z \triangleq \mathrm{col}\{z_1,z_2,\ldots,z_N\} \in \mathbb{R}^{Nn} | (\mathcal{L}\otimes I_n)Z = 0\right\}. \tag{2}$$

From Lemma 1, it follows that consensus is achieved if the states of agents stay on the consensus manifold, that is, $\|x_i - x_j\| = 0$ for all $i,j \in \{1,2,\ldots,N\}$ if $\mathrm{col}\{x_1,x_2,\ldots,x_N\} \in \mathcal{Z}$. Then, the optimal consensus problem to be solved for $\tau \in [t_k, t_k+\mathcal{T})$, $k=0,1,2,\ldots$, is formulated as follows.

Problem 1. At any update time t_k, for every agent $i=1,\ldots,N$, given $x_i(t_k)$, solve

$$\begin{aligned}
&\min_{u_i(\tau|t_k), z_i(\tau|t_k)} J(u_i(\tau|t_k), z_i(\tau|t_k))\\
&s.t.\ x_i(\tau+1|t_k) = Ax_i(\tau|t_k) + Bu_i(\tau|t_k),\ \tau \in [t_k, t_k+\mathcal{T}-1],\\
&\quad x_i(t_k|t_k) = x_i(t_k),\\
&\quad Z(\tau|t_k) \in \mathcal{Z},\ \tau \in [t_k, t_k+\mathcal{T}],
\end{aligned}$$

where $Z(\tau|t_k) = \mathrm{col}\{z_1(\tau|t_k), z_2(\tau|t_k),\ldots,z_N(\tau|t_k)\}$ is an auxiliary vector of decision variables defined on the consensus manifold; the cost function with given weighted matrices $Q_{i\mathcal{T}} \geq 0, Q_i \geq 0, R_i > 0$, is defined as

$$\begin{aligned}
J(u_i(\tau|t_k), z_i(\tau|t_k)) = \sum_{i=1}^{N}\Bigg\{\sum_{\tau=t_k}^{t_k+\mathcal{T}-1}\Big[&\| x_i(\tau|t_k)\\
&- z_i(\tau|t_k)\|_{Q_i}^2 + \|u_i(\tau|t_k)\|_{R_i}^2\Big]\\
&+ \|x_i(t_k+\mathcal{T}|t_k) - z_i(t_k+\mathcal{T}|t_k)\|_{Q_{i\mathcal{T}}}^2\Bigg\},
\end{aligned} \tag{3}$$

in which $\mathcal{T}$ is the prediction horizon and $\Delta t = t_{k+1} - t_k$ is the control period, satisfying $0 < \Delta t \leq \mathcal{T}$, and $k = 0,1,2,\ldots$. This quadratic cost function indicates the overall energy of the consensus error signal and the control input signal over the prediction internal, used as a performance index to measure the transient performance with a free consensus endpoint.

Applying the DMPC scheme, Problem 1 can be solved by the distributed algorithm (Algorithm 2) proposed in Wang, Duan et al. (2019) and the first portion of the optimal solution $u_i^{\star}(\tau|t_k)$, $\tau \in [t_k, t_{k+1})$ can be implemented as the real-time input. Following this pattern, the detailed DMPC algorithm to achieve linear quadratic optimal consensus is presented in Algorithm 1.

Remark 1. In the cost function (3), not only the control input sequence $u_i(\tau|t_k)$ but also the consensus manifold $Z(\tau|t_k)$ are regarded as decision variables used to optimize both the transient and the steady-state responses. Compared to the case with a fixed consensus trajectory (Ferrari-Trecate et al., 2009; Gao et al., 2017), i.e., $Z(\tau|t_k) = Z^{fix}(\tau|t_k) \in \mathcal{Z}$, the agents here will determine the optimal consensus point $Z^\star(\tau|t_k)$ according to the actual operation situation. From the principle of optimality, it can be concluded that $J(u_i^\star(\tau|t_k), Z^\star(\tau|t_k)) \le J(u_i^\star(\tau|t_k), Z^{fix}(\tau|t_k))$ for $Z^{fix}(\tau|t_k) \in \mathcal{Z}$. Therefore, the proposed mechanism leads to better consensus performance in general.

Remark 2. In Problem 1, finite-time optimization is solved because of real computation limits. In order to achieve steady state consensus, the MPC technique is utilized to extend the finite-time control sequence. However, MPC is a repeated open-loop optimal control method. Whether the MPC controller can stabilize the closed-loop system remains a question, which will be discussed next.

3. Stability analysis

The stability of the closed-loop system based on the proposed MPC algorithm is analyzed in this section.

Theorem 1. *For the multi-agent system (1) with an undirected and connected communication topology, if the weighted matrices* $Q_{i\mathcal{T}} \ge 0, Q_i > 0, R_i > 0$, *and there exist matrices* K_i *such that*

$$\Pi_i \le 0, \quad i \in \{1,2,\dots,N\}, \tag{4}$$

where $\Pi_i = \sum_{m=0}^{\Delta t-1} \|(A+BK_i)^m\|^2_{Q_i+K_i^T R_i K_i} + \|(A+BK_i)^{\Delta t}\|_{Q_{i\mathcal{T}}} - Q_{i\mathcal{T}}$,

then the consensus can be achieved asymptotically by the DMPC algorithm, that is, as $t_k \to \infty$, $\|x_i(t_k) - x_j(t_k)\| \to 0$ *for all* $i, j \in \{1,2,\dots,N\}$.

Proof. With the optimal solution $u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)$ of Problem 1 for $\tau \in [t_k, t_k+\mathcal{T})$ at sampled time t_k, construct a feasible control input at time t_{k+1}, inspired by Gao et al. (2017), as follows:

$$\hat{u}_i(\tau|t_{k+1}) = \begin{cases} u_i^\star(\tau|t_k), & \tau \in [t_{k+1}, t_k+\mathcal{T}), \\ u_i^{k+1}(\tau|t_{k+1}), & \tau \in [t_k+\mathcal{T}, t_{k+1}+\mathcal{T}), \end{cases} \tag{5}$$

and a feasible consensus state as

$$\hat{z}_i(\tau|t_{k+1}) = \begin{cases} z_i^\star(\tau|t_k), & \tau \in [t_{k+1}, t_k+\mathcal{T}], \\ A\hat{z}_i(\tau-1|t_{k+1}), & \tau \in [t_k+\mathcal{T}+1, t_{k+1}+\mathcal{T}], \end{cases} \tag{6}$$

which satisfies the constraints stated in Problem 1. Denoting the state associating with the feasible control input (5) along with the dynamics (1) from the initialization $\hat{x}_i(t_{k+1}|t_{k+1}) = x_i^\star(t_{k+1}|t_k)$ as $\hat{x}_i(\tau|t_{k+1})$, and the optimal solution at update time t_{k+1} as $u_i^\star(\tau|t_{k+1}), z_i^\star(\tau|t_{k+1})$ for $\tau \in [t_{k+1}, t_{k+1}+\mathcal{T})$, one has

$$\begin{aligned} & J(u_i^\star(\tau|t_{k+1}), z_i^\star(\tau|t_{k+1})) - J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)) \\ \le & J(\hat{u}_i(\tau|t_{k+1}), \hat{z}_i(\tau|t_{k+1})) - J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)) \\ = & \sum_{i=1}^N \Bigg\{ \sum_{\tau=t_{k+1}}^{t_{k+1}+\mathcal{T}-1} \Big[\|\hat{e}_i(\tau|t_{k+1})\|^2_{Q_i} + \|\hat{u}_i(\tau|t_{k+1})\|^2_{R_i} \Big] \\ & + \|\hat{e}_i(t_{k+1}+\mathcal{T}|t_{k+1})\|^2_{Q_{i\mathcal{T}}} - \sum_{\tau=t_k}^{t_k+\mathcal{T}-1} \Big[\|e_i^\star(\tau|t_k)\|^2_{Q_i} \\ & + \|u_i^\star(\tau|t_k)\|^2_{R_i} \Big] - \|e_i^\star(t_k+\mathcal{T}|t_k)\|^2_{Q_{i\mathcal{T}}} \Bigg\}, \end{aligned} \tag{7}$$

where $\hat{e}_i(\tau|t_{k+1}) = \hat{x}_i(\tau|t_{k+1}) - \hat{z}_i(\tau|t_{k+1})$ and $e_i^\star(\tau|t_k) = x_i^\star(\tau|t_k) - z_i^\star(\tau|t_k)$. Design the terminal controller of (5) as $u_i^{k+1}(\tau|t_{k+1}) = K_i\hat{e}_i(\tau|t_{k+1})$ for all $\tau \in [t_k+\mathcal{T}, t_{k+1}+\mathcal{T})$. Then, it follows from (1) that

$$\hat{e}_i(t_k+\mathcal{T}+m|t_{k+1}) = (A+BK_i)^m \hat{e}_i(t_k+\mathcal{T}|t_{k+1}), \tag{8}$$

for $m \ge 0$. Substituting (8) into (7) yields

$$\begin{aligned} & J(u_i^\star(\tau|t_{k+1}), z_i^\star(\tau|t_{k+1})) - J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)) \\ \le & \sum_{i=1}^N \Bigg\{ \sum_{\tau=t_k+\mathcal{T}}^{t_{k+1}+\mathcal{T}-1} \Big[\|\hat{e}_i(\tau|t_{k+1})\|^2_{Q_i} + \|\hat{u}_i(\tau|t_{k+1})\|^2_{R_i} \Big] \\ & + \|\hat{e}_i(t_{k+1}+\mathcal{T}|t_{k+1})\|^2_{Q_{i\mathcal{T}}} - \sum_{\tau=t_k}^{t_{k+1}-1} \Big[\|e_i^\star(\tau|t_k)\|^2_{Q_i} \\ & + \|u_i^\star(\tau|t_k)\|^2_{R_i} \Big] - \|e_i^\star(t_k+\mathcal{T}|t_k)\|^2_{Q_{i\mathcal{T}}} \Bigg\} \\ = & \sum_{i=1}^N \Bigg\{ \sum_{m=0}^{\Delta t-1} \|(A+BK_i)^m \hat{e}_i(t_k+\mathcal{T}|t_{k+1})\|^2_{Q_i+K_i^T R_i K_i} \\ & + \|(A+BK_i)^{\Delta t}\hat{e}_i(t_k+\mathcal{T}|t_{k+1})\|^2_{Q_{i\mathcal{T}}} \\ & - \sum_{\tau=t_k}^{t_{k+1}-1} \Big[\|e_i^\star(\tau|t_k)\|^2_{Q_i} + \|u_i^\star(\tau|t_k)\|^2_{R_i} \Big] \\ & - \|e_i^\star(t_k+\mathcal{T}|t_k)\|^2_{Q_{i\mathcal{T}}} \Bigg\}. \end{aligned} \tag{9}$$

With the first portion of the feasible controller (5), one gets $\hat{x}_i(t_k+\mathcal{T}|t_{k+1}) = x_i^\star(t_k+\mathcal{T}|t_k)$, $\hat{z}_i(t_k+\mathcal{T}|t_{k+1}) = z_i^\star(t_k+\mathcal{T}|t_k)$, and (9) gives

$$\begin{aligned} & \sum_{m=0}^{\Delta t-1} \|(A+BK_i)^m \hat{e}_i(t_k+\mathcal{T}|t_{k+1})\|^2_{Q_i+K_i^T R_i K_i} \\ & + \|(A+BK_i)^{\Delta t}\hat{e}_i(t_k+\mathcal{T}|t_{k+1})\|^2_{Q_{i\mathcal{T}}} \\ & - \|e_i^\star(t_k+\mathcal{T}|t_k)\|^2_{Q_{i\mathcal{T}}} \\ \le & \|x_i^\star(t_k+\mathcal{T}|t_{k+1}) - z_i^\star(t_k+\mathcal{T}|t_{k+1})\|^2_{\Pi_i} \\ \le & 0, \end{aligned} \tag{10}$$

where the last inequality is guaranteed by the constraints (4) on Π_i. Substituting (10) into (9), one concludes that

$$\begin{aligned} & J(u_i^\star(\tau|t_{k+1}), z_i^\star(\tau|t_{k+1})) - J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)) \\ \le & -\sum_{i=1}^N \Bigg\{ \sum_{\tau=t_k}^{t_{k+1}-1} \Big[\|e_i^\star(\tau|t_k)\|^2_{Q_i} + \|u_i^\star(\tau|t_k)\|^2_{R_i} \Big] \Bigg\} \\ \le & 0, \end{aligned} \tag{11}$$

which indicates that $\{J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)), k = 0,1,2,\dots\}$ is non-increasing. Therefore, $e_i^\star(\tau|t_k)$ and $u_i^\star(\tau|t_k)$ are bounded for all t_k. From Proposition 1.10 in Lasalle (1986), it follows that the positive limit set Ω, which collects all positive limit points of $\mathrm{col}\{e_i^\star(\tau|t_k), u_i^\star(\tau|t_k)\}$ as $t_k \to \infty$, is a non-empty, compact, positively invariant set. Since $J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)) \ge 0$, it follows that $\big\{J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)),\ k = 0,1,2,\dots\big\}$ is convergent as $t_k \to \infty$, i.e., $J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)) \to c$. For any element $\Omega_i \in \Omega$, there is a subsequence $\{t_{ki}\}$ such that $\mathrm{col}\{e_i^\star(\tau|t_{ki}), u_i^\star(\tau|t_{ki})\} \to \Omega_i$ and $J(\Omega_i) = c$. Combining the fact that Ω is positively invariant, it can be concluded that $\Omega \subset \Big\{u_i^\star(\tau|t_k), x_i^\star(\tau|t_k), z_i^\star(\tau|t_k) : J(u_i^\star(\tau|t_{k+1}), z_i^\star(\tau|t_{k+1})) - J(u_i^\star(\tau|t_k), z_i^\star(\tau|t_k)) = 0\Big\}$. Then, from the inequality (11), it is easy to get that $\Omega \subset \Big\{ u_i^\star(\tau|t_k), x_i^\star(\tau|t_k), z_i^\star(\tau|t_k) : \|e_i^\star(\tau|t_k)\|^2_{Q_i} = 0, \|u_i^\star(\tau|t_k)\|^2_{R_i} = 0, \tau \in [t_k, t_{k+1}), i \in \{1,2,\dots,N\} \Big\}$, which implies that $\|e_i^\star(\tau|t_k)\| \to 0$ as $t_k \to \infty$ for all $\tau \in [t_k, t_{k+1})$ and $i \in \{1,2,\dots,N\}$ since $Q_i > 0$. From the MPC scheme, it follows that the optimal solution $x_i^\star(\tau|t_k)$ for $\tau \in [t_k, t_{k+1})$ is the real-time state and $\mathrm{col}\{z_1^\star(\tau|t_k), z_2^\star(\tau|t_k), \dots,$

$z_N^\star(\tau|t_k)\} \in \mathcal{Z}$. Therefore, the consensus of multi-agent system is achieved asymptotically by using the DMPC algorithm. □

Remark 3. The linear quadratic consensus problem of multi-agent systems is solved distributedly in Wang, Duan et al. (2019) by leveraging connections to the alternating direction method of multiplier, in which finite-time control horizon is considered, that is, only transient performance is optimized. As for the steady-state response of the linear quadratic consensus problem, there is still no affirmative result. Therefore, the DMPC scheme is applied in this paper to extend the finite-time control sequence to the infinite horizon, with a condition of asymptotic consensus derived.

Remark 4. In the above stability analysis, the feasible consensus state (6) at update time t_{k+1} is constructed from the optimal solution $Z^\star(\tau|t_k)$ at the previous predictive step, which satisfies the consensus constraint $(\mathcal{L}\otimes I_n)Z^\star(\tau|t_k) = 0, \tau \in [t_{k+1}, t_k + \mathcal{T}]$, in Problem 1. From the dynamics (6), it is easy to get that $\hat{Z}(\tau|t_{k+1}) = \left(I_n \otimes A^{\tau - t_k - \mathcal{T}}\right)Z^\star(t_k+\mathcal{T}|t_k)$ for $\tau \in [t_k+\mathcal{T}+1, t_{k+1}+\mathcal{T}]$, which implies that the consensus constraint is satisfied for $\tau \in [t_k+\mathcal{T}+1, t_{k+1}+\mathcal{T}]$ since $\left(\mathcal{L}\otimes I_n\right)\hat{Z}(\tau|t_{k+1}) = \left(I_n \otimes A^{\tau-t_k-\mathcal{T}}\right)\left(\mathcal{L}\otimes I_n\right)Z^\star(t_k+\mathcal{T}|t_k) = 0$. Therefore, $\hat{Z}(\tau|t_{k+1})$ defined in (6) is always feasible, i.e., $\hat{Z}(\tau|t_{k+1}) \in \mathcal{Z}, \tau \in [t_{k+1}, t_{k+1}+\mathcal{T}]$. Then, the consensus error of agent i can be formulated as $\hat{x}_i(\tau|t_{k+1}) - \hat{z}_i(\tau|t_{k+1})$ without using the information of the whole network, so that the stability condition (4) is formulated in a decentralized form, which can be determined by each agent individually.

Remark 5. Referring to inequality (11), the weighted matrix Q_i is set to be positive definite for state consensus. Otherwise, only part of the state x_i will achieve consensus if Q_i is positive semi-definite, which opens the door to investigating the optimal output consensus problem.

Remark 6. In the DMPC scheme, the processing model is used to forecast system dynamics for controller design, where the prediction accuracy has a great influence on the control performance. Therefore, the proposed DMPC algorithm is more applicable for periodic systems and slowly varying systems than fast dynamic systems.

A sufficient condition to guarantee asymptotical consensus of the closed-loop multi-agent system is established in Theorem 1, which provides some theoretical guideline for the choice of weighted matrices $Q_i, Q_{i\mathcal{T}}, R_i$ and control period Δt. However, it is not easy to determine whether there exists a K_i satisfying the stability condition (4), which brings difficulties in choosing the weighted matrices in the DMPC algorithm. Therefore, the stability condition (4) is relaxed to a simpler one in the following corollary.

Corollary 1. *For the multi-agent system (1) with an undirected and connected communication topology, assume that (A, B) is controllable, $\Delta t \geq n$, and there exist $Q_{i\mathcal{T}} \geq 0, Q_i > 0, R_i > 0$, K_{0i}, satisfying*

$$\sum_{l=0}^{n-1} \|(A+BK_{0i})^l\|^2_{Q_i + K_{0i}{}^T R_i K_{0i}} - Q_{i\mathcal{T}} \leq 0, \tag{12}$$

for all $i \in \{1, 2, \ldots, N\}$, where K_{0i} is a feedback gain matrix assigning the closed-loop poles of system (1) to zero. Then, asymptotical consensus can be achieved by the DMPC algorithm.

Proof. According to the linear systems theory (Ogata, 1995), the closed-loop poles of system (1) can be placed arbitrarily by state feedback control if (A, B) is controllable. Therefore, there exists a feedback gain matrix K_{0i} making all eigenvalues of matrix $A+BK_{0i}$ be zero, which can be transformed into a Jordan form as follows:

$$F^{-1}(A+BK_{0i})F = \mathrm{diag}\{L_1, L_2, \ldots, L_q\}, \tag{13}$$

where F is a nonsingular transformation matrix and $L_i \in \mathbb{R}^{l_i \times l_i}$ are Jordan blocks corresponding to zero eigenvalues of the form $L_i = \begin{bmatrix} 0 & I_{l_i - 1} \\ 0 & 0 \end{bmatrix}$. Notice that the Jordan block L_i is a nilpotent matrix of degree at most n, so $F^{-1}(A+BK_{0i})^{\Delta t}F = 0$ holds for all $\Delta t \geq n$, which is equivalent to $(A+BK_{0i})^{\Delta t} = 0$. Then, for $\Delta t \geq n$, the stability condition (4) becomes

$$\sum_{l=0}^{n-1} \|(A+BK_{0i})^l\|^2_{Q_i + K_{0i}{}^T R_i K_{0i}} - Q_{i\mathcal{T}} \leq 0, \tag{14}$$

which is in accordance with (12). □

Algorithm 1 DMPC-based Linear Quadratic Optimal Consensus Algorithm

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Require: At each update time $t_k = 0, \Delta t, 2\Delta t, \cdots$, initialize $x_i^0(\tau|t_k) = x_i(t_k)$, $u_i^0(\tau|t_k) = \mathbf{0}$, $z_i^0(\tau|t_k) = x_i(t_k)$, $\lambda_i^0(\tau|t_k) = \mathbf{0}$, $\rho > 0$, $G_i > 0$, $H_i > \left(2\rho l_{ii} + L_{\delta i}\right)I_n$, $q = 0$, for every agent $i = 1, \cdots, N$, with $\tau \in [t_k, t_k+T]$. Set the stop condition $N_q > 0$. For subsystem $i \in \{1, 2, \cdots, N\}$, do in parallel:

repeat
  for $\tau = t_k + T - 1$ to t_k **do**
    Compute the control input $u_i^{q+1}(\tau|t_k) = -U_i(\tau)\left[V_i(\tau)x_i^{q+1}(\tau|t_k) - W_i(\tau)\right]$, where $U_i(\tau)$, $V_i(\tau)$ and $W_i(\tau)$ are gain matrices computed by individual agent independently through dynamic programming technique (refer to Theorem 3 in Wang, Duan et al. (2019) for more details).
  end for
  Update $x_i^{q+1}(\tau|t_k),\ \tau \in [t_k, t_k+T]$ from (1);
  Update $z_i^{q+1}(\tau|t_k),\ \tau \in [t_k, t_k+T-1]$ and $z_i^{q+1}(t_k+T|t_k)$ with communication:

$$z_i^{q+1}(\tau|t_k) = 2H_i^{-1}Q_i e_{xi}^q(\tau|t_k) + e_{ij}^q(\tau|t_k),$$

$$z_i^{q+1}(t_k+T|t_k) = 2H_i^{-1}Q_{i\mathcal{T}} e_{xi}^q(t_k+T|t_k) + e_{ij}^q(t_k+T|t_k),$$

  where $e_{xi}^q(\tau|t_k) = x_i^{q+1}(\tau|t_k) - z_i^q(\tau|t_k)$, $e_{ij}^q(\tau|t_k) = z_i^q(\tau|t_k) - \sum_{j=1}^N a_{ij}H_i^{-1}\left[\rho z_i^q(\tau|t_k) - \rho z_j^q(\tau|t_k) + \lambda_i^q(\tau|t_k) - \lambda_j^q(\tau|t_k)\right]$ for $\tau \in [t_k, t_k+T]$;
  Update the Lagrangian multiplier $\lambda_i^{q+1}(\tau|t_k) = \lambda_i^q(\tau|t_k) + \rho z_i^{q+1}(\tau|t_k),\ \tau \in [t_k, t_k+T]$;
  Set $q = q + 1$;
until $q > N_q$;
return the optimal solution $u_i^\star(\tau|t_k) = u_i^q(\tau|t_k), z_i^\star(\tau|t_k) = z_i^q(\tau|t_k)$ for $\tau \in [t_k, t_k+T)$, and select the first portion $u_i^\star(\tau|t_k), \tau \in [t_k, t_{k+1})$, as the real-time input.

Remark 7. It seems that the consensus condition in Corollary 1 is stricter than that in Theorem 1 because of the extra pole placement requirement on K_{0i}. Nevertheless, the coupling of K_i and $Q_{i\mathcal{T}}$ in (4) is removed by selecting such $K_i = K_{0i}$, which is easy to realize for controllable systems. Invoking the Geršgorin disk theorem (Horn & Johnson, 2012), the decoupled condition (12) can be satisfied easily by choosing the weighted matrix $Q_{i\mathcal{T}}$ diagonally dominant with the diagonal elements much greater than the elements of Q_i, R_i and K_{0i}. This is reasonable because $Q_{i\mathcal{T}}$ is the weight on the final consensus error. Note that the control sequence implemented in a real system is the optimal solution

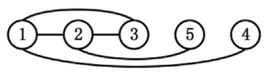

Fig. 1. The communication topology of a five-agent system.

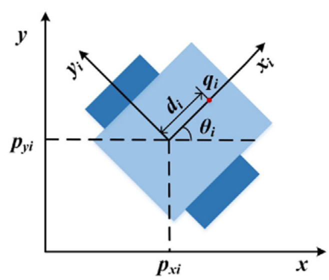

Fig. 2. Schematic of the ith wheeled vehicle.

of Problem 1, calculated by Algorithm 1, therefore the selection of K_{0i} does not influence the control performance of the resulting system.

For system with single input, the following example gives a specific method for parameter selection.

Example 1. Consider a single input multi-agent system of the controllable canonical form, with the system matrices in (1) as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & & \ddots & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix} \in \mathbb{R}^{n\times n}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \in \mathbb{R}^n. \quad (15)$$

By designing a feedback gain matrix as $K_{0i} = [a_0, a_1, \ldots, a_{n-1}]$, there is $A + BK_{0i} = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}$. Selecting the weighted matrices as $Q_i = \text{diag}\{q_i, q_i, \ldots, q_i\}$, $Q_{i\mathcal{T}} = \text{diag}\{2q_i, 3q_i, \ldots, (n+1)q_i\}$ and q_i much greater than R_i and $\|a_j\|, j = 0, 1, \ldots, n-1$, the condition (12) is satisfied since

$$\begin{aligned} &\sum_{l=0}^{n-1} \|(A + BK_{0i})^l\|^2_{Q_i + K_{0i}^T R_i K_{0i}} - Q_{i\mathcal{T}} \\ =&\sum_{l=0}^{n-1} \left\| \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}^l \right\|^2_{Q_i + K_{0i}{}^T R_i K_{0i}} - Q_{i\mathcal{T}} \\ =&\Big[l_{pq}\Big]_{n\times n} : l_{pq} = \begin{cases} -q_i + \sum_{j=1}^{p} R_i a_{j-1}^2, & p = q, \\ \sum_{j=1}^{p} a_{p-j} a_{q-j}, & p < q, \\ l_{qp}, & p > q, \end{cases} \\ \leq& 0, \end{aligned} \quad (16)$$

where the last inequality is derived from the Geršgorin disk theorem (Horn & Johnson, 2012).

4. Simulation results on formation control

The proposed MPC algorithm is applied to solve the formation control problem of multi-agent systems in this section. Consider a network of five wheeled vehicles with the communication topology as shown in Fig. 1.

Let p_{xi}, p_{yi} and θ_i denote the Cartesian position and orientation of the ith vehicle, as shown in Fig. 2, with its dynamics described by

$$\begin{aligned} &\dot{p}_{xi} = v_i \cos\theta_i,\ \dot{p}_{yi} = v_i \sin\theta_i,\ \dot{\theta}_i = \omega_i, \\ &\dot{v}_i = F_i/m_i,\ \dot{\omega}_i = M_i/J_i, \end{aligned} \quad (17)$$

where v_i and ω_i are linear and angular velocities; m_i and J_i are mass and moment of inertia; F_i and τ_i are applied force and torque, respectively. Referring to Ren and Beard (2008), the dynamic equations (17) can be feedback linearized at a fixed reference point $q_i = [q_{xi}, q_{yi}]^T$ off the center of the vehicle, where

$$q_{xi} = p_{xi} + d_i \cos\theta_i,\ q_{yi} = p_{yi} + d_i \sin\theta_i. \quad (18)$$

With the reference control input defined by $u_i = [u_{1i},\ u_{2i}]^T$, $u_{1i} = \frac{F_i}{m_i}\cos\theta_i - \frac{d_i M_i}{J_i}\sin\theta_i - v_i\omega_i \sin\theta_i - d_i\omega_i^2\cos\theta_i$, $u_{2i} = \frac{F_i}{m_i}\sin\theta_i + \frac{d_i M_i}{J_i}\cos\theta_i + v_i\omega_i\cos\theta_i - d_i\omega_i^2\sin\theta_i$, the linearized dynamic equation of the ith vehicle can be formulated as

$$\ddot{q}_i(t) = u_i(t). \quad (19)$$

Referring to the numerical control method developed in Ogata (1995), the discretization form of (19) is obtained as

$$\begin{bmatrix} q_i(\tau+1) \\ \dot{q}_i(\tau+1) \end{bmatrix} = \begin{bmatrix} I_2 & \delta I_2 \\ 0 & I_2 \end{bmatrix} \begin{bmatrix} q_i(\tau) \\ \dot{q}_i(\tau) \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\delta^2 I_2 \\ \delta I_2 \end{bmatrix} u_i(\tau), \quad (20)$$

where $\dot{q}_i$ is the velocity of the reference point, δ is the sampled time and the state variable is $x_i = [q_i^T, \dot{q}_i^T]^T$. Based on the discretization model (20), the MPC sequence can be solved and then held by a zero-order holder to control the multi-vehicle system (17).

To achieve formation control, the cost function (3) is modified as $J = \sum_{i=1}^{N} \Big\{ \|x_i(t_k + \mathcal{T}|t_k) - z_i(t_k + \mathcal{T}|t_k) - x_{ci}\|^2_{Q_{i\mathcal{T}}} + \sum_{\tau=t_k}^{t_k+\mathcal{T}-1} \big[\|x_i(\tau|t_k) - z_i(\tau|t_k) - x_{ci}\|^2_{Q_i} + \|u_i(\tau|t_k)\|^2_{R_i} \big] \Big\}$, where x_{ci} denotes the relative position between agent i and the consensus point in the desired formation pattern. Parameters of the ith vehicle are set as $d_i = \text{rand}(0.115, 0.125)$ m, $m_i = \text{rand}(9.5, 10.5)$ kg, $J_i = \text{rand}(0.12, 0.14)$ kg m^2, where $\text{rand}(a, b)$ denotes a random number with a uniform distribution on the interval $[a, b]$. The sampled time, prediction horizon and control period are selected as $\delta = 0.6$ s, $T = 5$ and $\Delta t = 3$, respectively. The weighted matrices are chosen as $Q_i = \text{diag}\{5, 4\} \otimes I_2$, $Q_{i\mathcal{T}} = \text{diag}\{100, 60\} \otimes I_2$ and $R_i = \text{diag}\{3, 3\}$, which satisfy the consensus condition. The initial states of the ith vehicle are set as $p_{xi}(0) = \text{rand}(-3, 3)$ m, $p_{yi}(0) = \text{rand}(-3, 3)$ m, $v_i(0) = \text{rand}(-1, 1)$ m/s, $\omega_i(0) = \text{rand}(-0.2, 0.2)$ rad/s and $\theta_i(0) = \text{rand}(0, 2\pi)$. The initial iterative values at update time t_k are selected as $u_i^0(\tau|t_k) = [0, 0]^T$ and $z_i^0(\tau|t_k) = x_i(t_k)$. The desired formation pattern is defined as $[q_{c1}, q_{c2}, \ldots, q_{cN}] = [0, -0.3, 0.5, -0.5, 0.3; 0.5, -0.4, 0.2, 0.2, -0.4]$, which forms a regular pentagon.

The responses of the wheeled vehicle systems are recorded as shown in Figs. 3 to 5. From Fig. 3, it can be seen that each vehicle reaches the desired formation position and velocity rapidly. Moreover, Fig. 4 shows that the control inputs are damped quickly and satisfy the saturation constraints set in Ren and Beard (2008). The trajectories of formation are depicted in Fig. 5, which illustrates the efficacy of the DMPC algorithm.

For comparison, the static state-feedback control (SSFC) method (Li, Duan, Chen, & Huang, 2010) and the positively invariant terminal region-based DMPC (PITR-DMPC) method (Gao et al., 2017) are simulated to evaluate the effectiveness of Algorithm 1. Define the formation error cost function as $J_x(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\|x_i(t) - x_{ci} - x_j(t) - x_{cj}\|^2_{Q_i}$, the input cost function as $J_u(t) = \sum_{i=1}^{N} \|u_i(t)\|^2_{R_i}$, and the composite cost function as

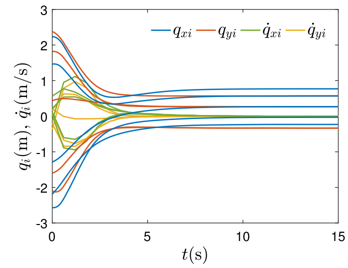

Fig. 3. Position and velocity curves of the reference point.

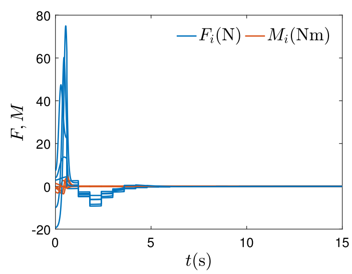

Fig. 4. Control inputs of each vehicle.

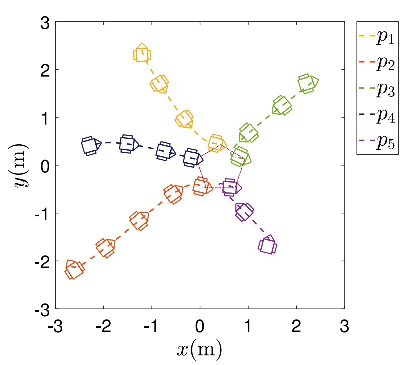

Fig. 5. The vehicle trajectories of formation.

$J_{sum}(t) = J_x(t) + J_u(t)$. The values of the cost functions generated by the different control methods are recorded in Figs. 6–7, from which it can be seen that the proposed scheme provides faster convergence rate, less overshoot and lower energy consumption.

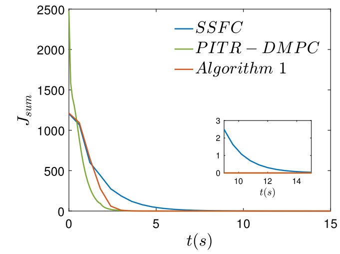

Fig. 7. Values of the composite cost function.

Note that although the PITR-DMPC method gets a faster convergence rate of the formation error, it requires overlarge control input in the initial stages, which is not reasonable in practice. Therefore, it is concluded that the proposed DMPC algorithm has overall better formation performances.

5. Conclusions

This paper studies the linear quadratic optimal consensus problem for discrete-time multi-agent systems. Based on the alternating direction method of multiplier algorithm proposed in Wang, Duan et al. (2019), a distributed model predictive control approach is developed to optimize the steady-state response in addition to the transient response of the consensus process. The stability of the closed-loop system is analyzed by appropriately constructing feasible solutions, which derives some constraints on the weighted matrices and sampled steps of all individual agents. For controllable systems, the consensus condition is relaxed for the convenience of parameter determination. The proposed algorithm is then applied to solving the formation control problem of multiple wheeled vehicles with numerical simulations, which demonstrate the efficiency of the proposed algorithm for performance optimization. In the future, heterogeneous and constrained dynamics will be addressed to extend the applicability of the proposed algorithm in practice.

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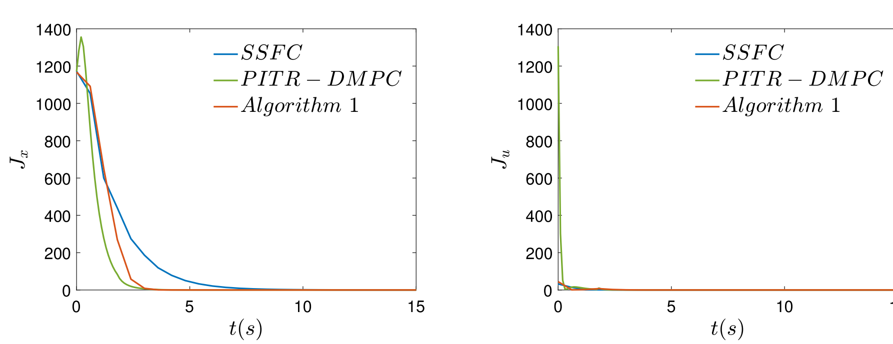

Fig. 6. Values of the formation error cost function and input cost function.

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