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Extended Kalman filter design to estimate the poisons concentrations in the P.W.R nuclear reactors based on the reactor power measurement

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ABSTRACT

An important problem in nuclear power plants is reactor power control. Considering the importance of the neutron absorber poisons such as xenon and samarium in design of the nuclear reactor control system and regarding the limitations of the xenon and samarium concentrations measurement, in this paper, an Extended Kalman Filter (E.K.F) is presented to estimate the xenon & samarium concentrations. Besides, Precursors produce delayed neutrons which are most important in specification of reactor period and control of nuclear reactor, but precursors densities cannot be measured directly and the designed Extended Kalman Filter can estimate delayed neutrons precursors densities. The reactor core is simulated based on the point kinetics equations and three delayed neutron groups.

Simulation results are presented to demonstrate the effectiveness of the proposed observer in terms of performance, robustness and stability and show that the Extended Kalman Filter (E.K.F) follows the actual system variables accurately and is satisfactory in the presence of the parameters uncertainties and disturbances.

1. Introduction

In nuclear reactors, some thermal fission products are important due to their large absorption cross-section for thermal neutrons. Therefore, these fragments are called neutron-absorber poisons which two more important of them are xenon135 (^{135}Xe) and samarium149 (^{149}Sm).

There exists a different behavior referred to radioactive decay; ^{149}Sm is stable isotope while ^{135}Xe decays with a half-life $T_{1/2} = 9.4(hours)$ to ^{135}Cs. So, oscillations of samarium are not same as xenon oscillations (Moreira, 2015). Control system for controlling reactor characteristics might measure the entire variables of the reactor system. But measurement of all of the variables such as xenon and samarium concentrations and delayed neutrons precursors densities is not possible. Therefore, these immeasurable variables must be estimated during control process and load-following operation.

Gabor et al. (2009) identified a model for VVER-type pressurized water reactor with temperature effects and xenon poisoning. They validated the model under varying load conditions. They also estimated some parameters of the plant with known data of some parameters like neutron density and coolant temperature. Wang peng et al. (2001), Butt and Bhatti (2008), Qadeer and Bhatti (2011) and Qaiser et al.(2009) estimated the external reactivity and Xenon concentration of nuclear reactor using sliding mode observer. They used point kinetic model of nuclear reactor for reactivity estimation and Cher nick's model for estimation of xenon concentration. The sliding mode observer exhibits acceptable tracking performance in the presence of parametric uncertainty only at the expense of high gains and control chattering. Indeed, an undesirable characteristic of this observer is chattering and its performance degradation in gaining robustness. Therefore, it seems that a high performance observer system is still needed for the load-following operation in the nuclear reactors.

Kalman filtering is a widely used method in many areas of signal processing, control, and opti-mization, e.g., adaptive filtering, estimation (Mendel, 1986), prediction, robust control, state observation (Misawa and Hedrick, 1989), and many others. Recently, Viegas et al. (2016) studied on the stability of the continuous-time Kalman filter subject to exponentially decaying perturbations.

In order to improve the performance of the Kalman filter application on the nonlinear systems, the Extended Kalman Filter (E.K.F) has been developed. Extended Kalman filter (E.K.F) is an optimal observer method for nonlinear systems with statistics noise of measurement and processes and has been used as a widely used method in many areas of signal processing, control, and optimization. (Lewis et al., 2008a,b). many researchs and articles has been

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presented about usefully and successfully of the E.K.F application (Misawa and Hedrick, 1989; Chang and Tabaczyinski, 1984). Considering the statistical behavior in the nuclear reactors and in order to the robustness against the statistics noise of measurement and processes, application of E.K.F in nuclear reactors is necessary and useful.

The goal of this paper is to present the Extended Kalman Filter observer system for nuclear reactors based on the skinner-cohen reactor model with reactivity feedback due to temperature and poisons (xenon and samarium) concentrations to estimate the xenon & samarium concentrations and delayed neutrons precursors densities.

The simulation results demonstrate the effectiveness of the proposed observer system in the presence of the parameters uncertainties and disturbances. Also, the comparison between the designed Kalman Filter, Extended Kalman Filter and the linear Luenberger observer has been done which shows a significant improvement in the actual system variables tracking and an increased ability in disturbance rejection for E.K.F.

2. The reactor core model

In this paper the nuclear reactor core has been simulated based on the point kinetics nuclear reactor model with three groups of delay neutrons based on the Skinner-Cohen model which has been validated and benched (Hetrick, 1965). Indeed, Point Kinetic Equations (PKE) are valid and accurate when there are not big neutron flux distortions in the reactor core. Therefore, in this paper, it was assumed that there are not big neutron flux distortions in the nuclear reactor core. The model assumes feedback from lumped fuel and coolant temperatures. Also, the effect of xenon and samarium concertions are included.

The normalized model, with respect to an equilibrium condition, based on point kinetics equations with three delayed neutron groups are as follows:

$$\begin{cases} \frac{dn_r}{dt} = \frac{\rho-\beta}{l} n_r + \sum_{i=1}^{3} \frac{\beta_i}{l} c_{ri} \\ \frac{dc_{ri}}{dt} = \lambda_i\, n_r - \lambda_i c_{ri} \qquad i = 1,2,3 \end{cases} \tag{1}$$

where n_r is neutron density relative to initial equilibrium density and C_{ri} is ith group precursor relative density normalized with the initial equilibrium density. The reactor power is displayed as follows:

$$P(t) = P_0 n_r \tag{2}$$

where P_0 is the nominal power (MW).Based on lumped fuel model, the thermal-hydraulics model of the reactor core is represented as follows (Hetrick, 1965):

$$\begin{cases} dT_f/dt = \frac{f_f P_0}{\mu_f} n_r - \frac{\Omega}{\mu_f} T_f + \frac{\Omega}{\mu_f} T_c \\ dT_c/dt = \frac{(1-f_f)P_0}{\mu_c} n_r + \frac{\Omega}{\mu_c} T_f - \left(\frac{2M+\Omega}{2\mu_c}\right)(2T_c - 290) + \left(\frac{2M-\Omega}{2\mu_c}\right) 290 \end{cases} \tag{3}$$

The equations related to the changes of the xenon and samarium concentrations are also as follows:

$$\begin{cases} \frac{dXe}{dt} = \gamma_{Xe} \Sigma_f \varphi + \lambda_I I - \sigma_X Xe.\varphi - \lambda_{Xe}.Xe \\ \frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I \end{cases} \tag{4}$$

$$\begin{cases} \frac{dSm}{dt} = \lambda_{pm} Pm - \sigma_{sm} Sm.\varphi \\ \frac{dPm}{dt} = \gamma_{pm} \Sigma_f \varphi - \lambda_{pm} Pm \end{cases} \tag{5}$$

Finally, the reactivity input and feedback to the point kinetics model are represented with the following equations

$$\rho = \rho_r + \rho_T + \rho_{Xe} + \rho_{Sm} \tag{6}$$

where

$$\frac{d\rho_r}{dt} = G_r Z_r \tag{7}$$

$$\rho_T = \alpha_f (T_f - T_{f_0}) + \alpha_c (T_c - T_{c_0}) \tag{8}$$

$$\rho_{Xe} = -\frac{\sigma_X (Xe - Xe_0)}{\Sigma_f} \tag{9}$$

$$\rho_{Sm} = -\frac{\sigma_{sm} (Sm - Sm_0)}{\Sigma_f} \tag{10}$$

All the parameters have been described in Table 1. Also, the parameter values of the reactor model are given in Table 2.

3. Preliminary

In this section, a brief description of the relevant theory and Kalman filter design algorithms used in the development of the PWR nuclear reactor observer is given (See Table 3).

3.1. Kalman filter

Kalman filter is based on the minimization of the mean-square error as the expected value of the Euclidean norm squared of estimation error; continues Kalman filter has been derived for continues linear systems (Lewis et al., 2008a,b);

Linear system has been modeled as follows (Lewis et al., 2008a, b):

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \tag{11}$$

$$\begin{cases} z(t) = Hx(t) + v(t) \\ x(0) \approx (\bar{x}_0, P_0), \quad w \approx (0, Q), v \approx (0, R) \end{cases} \tag{12}$$

where $w \approx (0, Q), v \approx (0, R)$ are white noises.

Table 1
Nomenclature.

Fraction of delayed fission neutrons	β
Effective precursor radioactive decay constant(1/s), chosen to match the three group reactor transfer function to a six delayed neutron group reactor transfer	λ_i
Effective prompt neutron lifetime (s)	l
Core reactivity	ρ
Reactivity change due to the control rod	ρ_r
Feedback reactivity change due to temperature changes	ρ_T
Feedback reactivity change due to xenon concentration variation,	ρ_x
Feedback reactivity change due to samarium concentration variation	ρ_{Sm}
Control rod speed (fraction of core length per second)	Z_r
Total reactivity of the rod	G_r
Macroscopic fission cross-section ($cm \wedge (-1)$)	Σ_f
Heat transfer coefficient between fuel and coolant ($MW.C^{-1}$)	Ω
Mass flow rate times heat capacity of the water ($MW.C^{-1}$)	M
Neutron flux ($n.cm^{-2}.s^{-1}$)	φ
fraction of reactor power deposited in the fuel	f_f
Total thermal capacity of the fuel and structural material ($MW.s.C^{-1}$)	μ_f
Total heat capacity of the reactor coolant($MW.s.C^{-1}$)	μ_c
Xenon decay constant (s^{-1})	λ_x
Iodine decay constant (s^{-1})	λ_I
Xenon yield	γ_x
Iodine yield	γ_I
Microscopic absorption cross-section of xenon (cm^2)	σ_x
Average reactor coolant temperature, fuel temperature (C)	T_c, T_f

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Table 2
Values of constants used for simulations.

Parameters	Value
Thermal power	2500 MW
Core high	400 cm
Core radius	200 cm
Diffusion constant (D)	0.16 cm
Mean velocity of thermal neutron (υ)	2200 m/s
Microscopic absorption cross section of Xenon (σ_X)	3.5×10^{-18} cm^2
Fractional fission yield of Xenon (γ_{Xe})	0.003
Fractional fission yield of Iodine (γ_I)	0.059
Decay constant of Xenon (λ_{Xe})	2.1×10^{-5} s^{-1}
Decay constant of Iodine (λ_I)	2.9×10^{-5} s^{-1}
Fractional fission yield of Promethium (γ_{pm})	1.08e-2
Decay constant of Promethium (λ_{pm})	3.6e-6 s^{-1}
Macroscopic fission cross section (Σ_f)	0.3358 cm ∧ (−1)
Total delayed neutron fraction (β)	0.0065
Delayed neutron fraction of first group neutron precursor (β_1)	0.00021
Delayed neutron fraction of second group neutron precursor (β_2)	0.00225
Delayed neutron fraction of third group neutron precursor (β_3)	0.00404
Radioactive decay constant of first group neutron precursor (λ_1)	0.0124 s^{-1}
Radioactive decay constant of second group neutron precursor (λ_2)	0.0369 s^{-1}
Radioactive decay constant of third group neutron precursor (λ_3)	0.632 s^{-1}
Total reactivity worth of control rod (G_r)	14.5×10^{-3}
Heat capacity of fuel (μ_f)	26.3 MW.s/°C
Total heat capacity of the reactor coolant (μ_c)	$(\frac{160}{9} n_{r0} + 540.022)Mw\frac{s}{°C}$
Heat transfer coefficient between fuel and coolant (Ω)	$(\frac{5}{3} n_{r0} + 4.93333)MW\frac{s}{°C}$
α_f	$(n_{r0} - 4.24) \times 10^{-5} \frac{\delta k}{k\, °C}$
α_c	$(-4n_{r0} - 17.3) \times 10^{-5} \frac{\delta k}{k\, °C}$
Fractional of heat energy reach the fuel from total heat produced in reactor core (f_f)	0.92
Effective prompt neutron lifetime (l)	2×10^{-5} s

Table 3
Kalman filter parameters.

Define model & Kalman filter Parameters	
x	State vector
u	Control signal
$Q = ww^t$	Covariance of the process noise
$R = \upsilon\upsilon^t$	Covariance of the measurement noise
w	Process noise
υ	Measurement noise
A, B, B$_w$, H	Matrix developed their parameters' on Transition in time of system state
$x(0)\bar{x}_0$	Initial system statemean initial system state
P_0	Covariance of initial system state
$\hat{x}$	State estimation
$\hat{x}_0$	initial state estimation
k	Kalman filter gain

The dynamics of the continuous-time Kalman filter for the above system is as follows (Lewis et al., 2008a,b):

$$\dot{\hat{x}} = A\hat{x} + Bu + K(z - H.\hat{x}) \tag{13}$$

where K is the Kalman gain as:

$$K(t) = P(t)H^T R^{-1} \tag{14}$$

Error covariance update is a Riccati equation as following:

$$\begin{cases} \dot{P}(t) = AP(t) + P(t)A^T + B_w Q B_w^T - P(t)H^T R^{-1} HP(t) \\ P(0) = P_0 \end{cases} \tag{15}$$

3.2. Extend Kalman filter (E.K.F)

The optimal estimator for linear stochastic systems with Gaussian statistics has been derived. It is known as the Kalman filter, and is itself a linear system with a gain matrix that depends on the solution to a matrix quadratic equation. The optimal estimation problem for nonlinear systems is in general very complicated, and only in a few special cases do algorithms exist that are easy to implement or understand. In order to improve the performance of the Kalman filter application on the nonlinear systems, the Extended Kalman Filter (E.K.F) has been developed. Extended Kalman Filter (E.K.F) is an optimal observer method for nonlinear systems with statistics noise of measurement and processes (Lewis et al., 2008a,b).

At the first, consider the following continuous nonlinear system:

$$\begin{cases} \dot{x} = f(x, u, t) + G(t)w \\ y = h(x, t) + \upsilon \end{cases} \tag{16}$$

where $w \approx (0, Q)$, $\upsilon \approx (0, R)$ are white noises and $x(0) \approx (\bar{x}_0, P_0)$. G(t) is the process noise matrix . The dynamics of the continuous-time Extended Kalman Filter (E.K.F) is as follows (Lewis et al., 2008a,b):

$$\begin{cases} \dot{\hat{x}} = f(\hat{x}, u, t) + K(t)(y - \hat{y}) \\ \hat{y} = h(\hat{x}, t) \end{cases} \tag{17}$$

where K is the Kalman gain as:

$$K(t) = P(t)H^T(\hat{x}, t)R^{-1} \tag{18}$$

Error covariance update is a Riccati equation as following:

$$\begin{cases} \dot{P}(t) = A(\hat{x}, t)P(t) + P(t)A^T(\hat{x}, t) + G(t)QG^T(t) - P(t)H^T(\hat{x}, t)R^{-1}H(\hat{x}, t)P(t) \\ P(0) = P_0 \\ \hat{x}(0) = \bar{x}_0 \end{cases} \tag{19}$$

where

$$A(\hat{x}, t) = \left.\frac{\partial f(x, u, t)}{\partial x}\right|_{\hat{x}(t)} \tag{20}$$

$$H(\hat{x}, t) = \frac{\partial h(x, t)}{\partial x}|_{\hat{x}(t)} \tag{21}$$

Indeed, error covariance and Kalman gain must be computed on-line in real time as the data become available. Therefore, E.K.F is more accurate than Kalman filter and is more suitable for non-linear systems.

4. Kalman filter design to estimate the poisons concentrations and precursors densities

Since the xenon & samarium concentrations and delayed neutron precursors densities cannot be measured in nuclear reactors, an observer is needed to estimate the immeasurable values. In this section, a Kalman filter observer is proposed based on the reactor power measurement. According to the point kinetics equations and observer structure (13), the reactor core power is used as an output and the immeasurable values are estimated as follows:

The nuclear reactor core model presented in Section 2 is nonlinear and can be presented as:

$$\begin{cases} \dot{X} = f(X, Z_r) + B_w w \\ Y = H.X + \upsilon \end{cases} \tag{22}$$

where

$$X = \begin{bmatrix} n_r \\ c_{r_1} \\ c_{r_2} \\ c_{r_3} \\ T_f \\ T_c \\ \rho_r \\ I \\ Xe \\ Pm \\ Sm \end{bmatrix} \tag{23}$$

Y = n_r and H = [1,0,0,0,0,0,0,0,0,0,0]. B_w is the process noise matrix and w is the process noise which contains the disturbance on the control rod speed (Z_r) and parametric uncertainties on the parameters: $\beta_i, \lambda_I, \lambda_{Xe}, \lambda_{Pm}, \lambda_{Sm}$.

$$B_w = \begin{bmatrix} \sum_{i=1}^{3}\beta_i \times C_{r_i}(t)/l - \beta \times n_r(t)/l & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & Gr \\ \lambda_I \times I(t) - \lambda_{xe} \times Xe(t) & 0 \\ -\lambda_I \times I(t) & 0 \\ \lambda_{pm} \times Pm(t) & 0 \\ -\lambda_{pm} \times Pm(t) & 0 \end{bmatrix} \tag{24}$$

Also, v is the white noise as the measurement noise. Considering the Kalman Filter structure (13), firstly, the presented nonlinear model is linearized about the operating condition (equilibrium point): $X\_\{eq\}, U\_\{eq\}$ where U_{eq} = Z_{r0} = 0, and then Kalman Filter structure is obtained as follows:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + BZ_r + K(t)(\delta n_r - H.\hat{x}) \\ K(t) = P(t)H^T R^{-1} \end{cases} \tag{25}$$

where

$$\hat{x} = \hat{X} - X_{eq}, \quad \delta n_r = n_r - n_{r0} \tag{26}$$

$$\begin{cases} \dot{P}(t) = AP(t) + P(t)A^T + B_w Q B_w^T - P(t)H^T R^{-1} HP(t) \\ Q = w^T w, \quad R = v^T v \\ P(0) = P0 \end{cases} \tag{27}$$

A and B, are states and input matrixes for linearized system, respectively as:

$$A = \left.\frac{\partial f}{\partial X}\right|_{X_{eq},U_{eq}}, B = \left.\frac{\partial f}{\partial U}\right|_{X_{eq},U_{eq}} \tag{28}$$

4.1. Simulation results of the Kalman filter

In this section, to evaluate the performance and robustness of the proposed observer structure a set of simulations is performed considering process noise, measurement noise and parameter uncertainties on the reactor model described in Section2.

The performance of the Kalman filter has been shown for 80% → 50% → 80% demand power level change. All system parameters are perturbed by ±20% from their nominal values. Precursor densities and poisons concentrations are estimated from Kalman filter equations described in Section 4. Real modeled and estimated relative neutron densities of reactor core have been shown in Fig. 1. Real modeled and estimated first, second and third groups of relative precursor densities of reactor core have been shown in Figs. 2–4, respectively. Real modeled and estimated iodine and xenon concentrations have been shown in Figs. 5 and 6, respectively. Real modeled and estimated Promethium and Samarium concentrations have been shown in Figs. 7 and 8, respectively. Results show that the modeled and estimated variables closely agree and the Kalman filter observer follows the actual system variables.

The results confirm the stability of Kalman filter system, but perfect tracking and good convergence to the actual states have not been achieved exactly; therefore, in the next section, E.K.F system has been designed and presented to improve the performance and tracing capability of Kalman filter.

5. Extended Kalman filter design to estimate the poisons concentrations and precursors densities

In order to improve the performance of the Kalman filter designed in the previous section, the Extended Kalman Filter

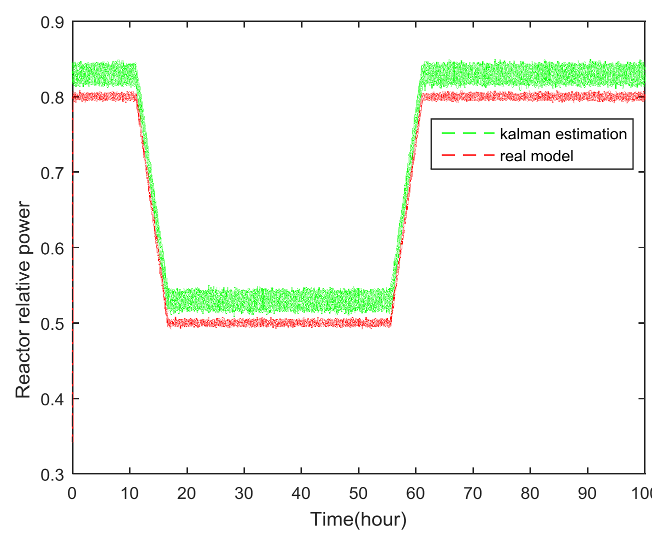

Fig. 1. Rreactor relative power, n_r.

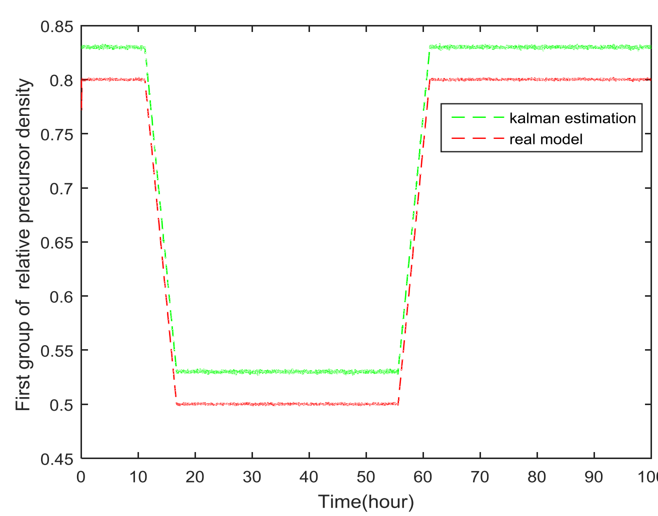

Fig. 2. First group of Relative Precursor density, C_{r1}.

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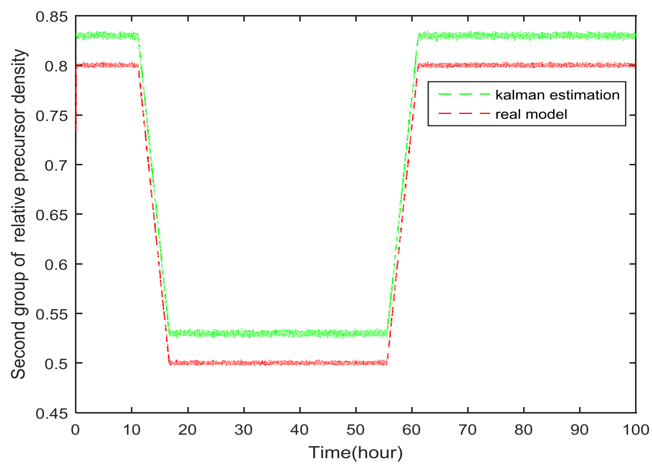

Fig. 3. Second group of Relative Precursor density, C_{r2}.

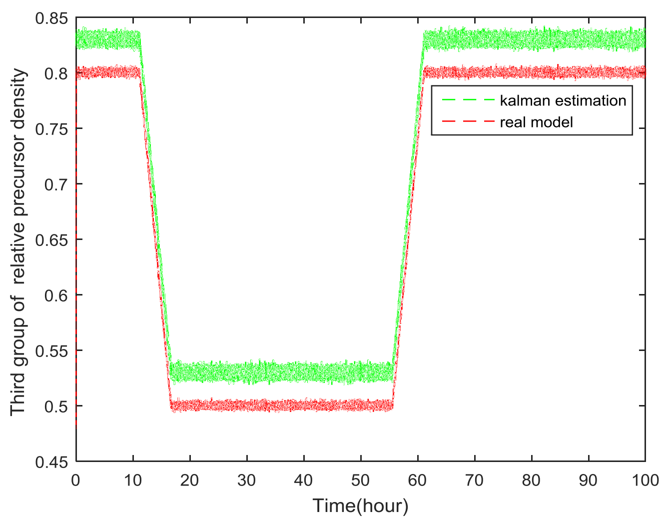

Fig. 4. Third group of Relative Precursor density, C_{r3}.

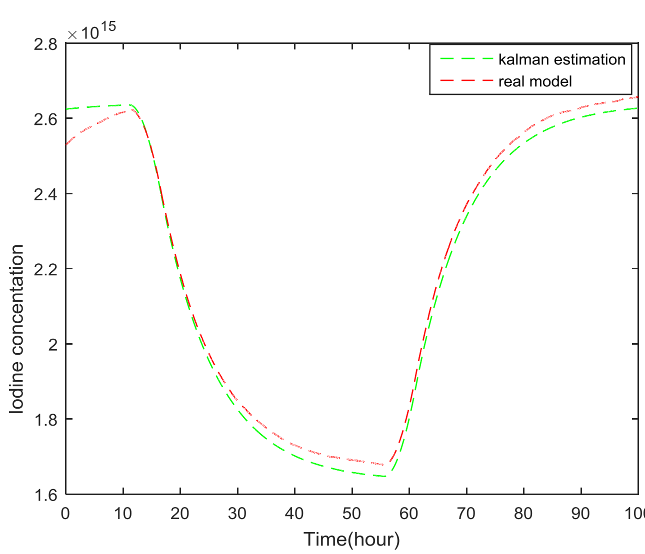

Fig. 5. Iodine concentration, *I*.

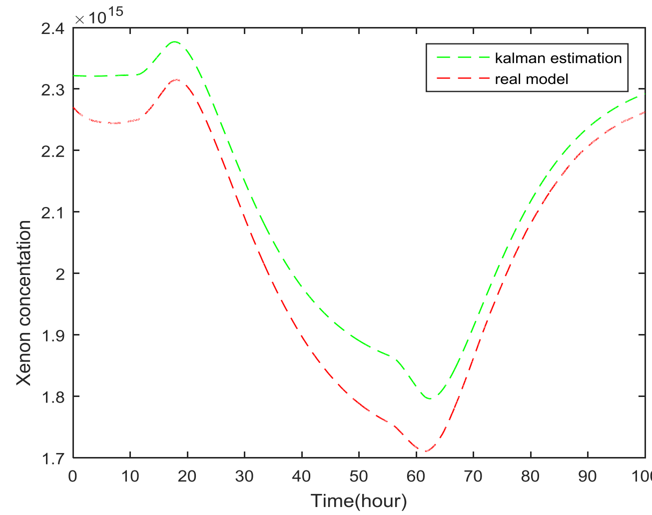

Fig. 6. Xenon concentration, *Xe*.

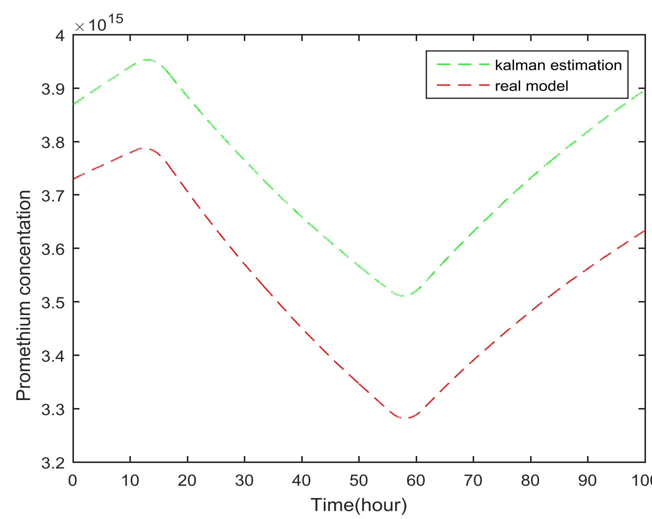

Fig. 7. Promethium concentration, Pm.

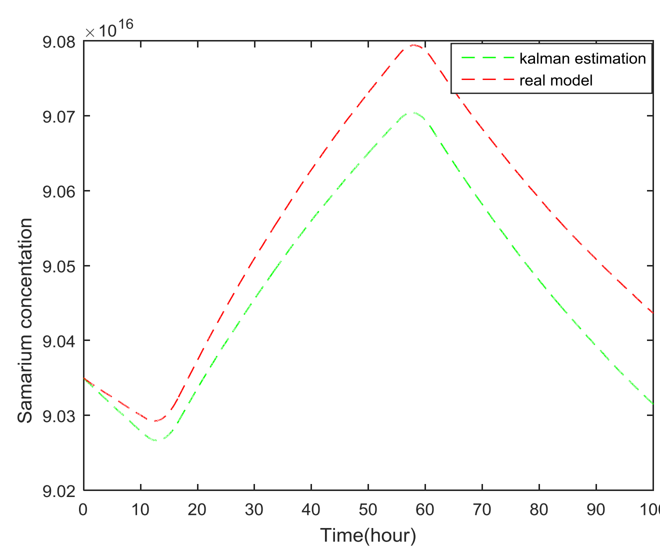

Fig. 8. Samarium concentration, Sm.

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(E.K.F) has been developed. According to the nonlinear point kinetics Eq. (22) and E.K.F structure (17), the reactor core power is used as an output and the immeasurable values are estimated as follows:

$$\begin{cases} \dot{\widehat{X}} = f(\widehat{X}, Z_r) + K(t)(Y - \widehat{Y}) \\ \widehat{Y} = H\widehat{X} \end{cases} \quad (29)$$

where K is the Kalman gain as:

$$K(t) = P(t)H^T R^{-1} \quad (30)$$

$$\widehat{Y} = \widehat{n}_r, \quad H = [1,0,0,0,0,0,0,0,0,0,0] \quad (31)$$

Also, $\widehat{X}$ is the estimation of the state vector (23).

Error covariance update is a Riccati equation as following:

$$\begin{cases} \dot{P}(t) = A(\widehat{X}, t)P(t) + P(t)A^T(\widehat{X}, t) + B_w Q B_w^T - P(t)H^T R^{(} - 1)HP(t) \\ P(0) = P_0 \\ X(0) = \bar{X}_0 \end{cases} \quad (32)$$

where

$$A(\widehat{X}, t) = \left.\frac{\partial f(X, Z_r)}{\partial X}\right|_{\hat{X}(t)} \quad (33)$$

Indeed, error covariance and Kalman gain are computed on-line in real time as the data become available. Therefore, E.K.F is more accurate than Kalman filter and is more suitable for the nuclear reactor model as a non-linear system. Based on the applicable inputs in nuclear power plants, designed E.K.F system is applied to the reactor and results are indicated in the next section.

5.1. Simulation results of the Extended Kalman filter

In this section, to evaluate the performance and robustness of the proposed Extended Kalman filter, a set of simulations is performed considering process noise, measurement noise and parameter uncertainties on the reactor model described in Section 2 as same as Section 4.1.

The performance of the Extended Kalman filter has been shown for 80% → 50% → 80% demand power level change. All system parameters are perturbed by ±20% from their nominal values. Precursor densities and poisons concentrations are estimated from

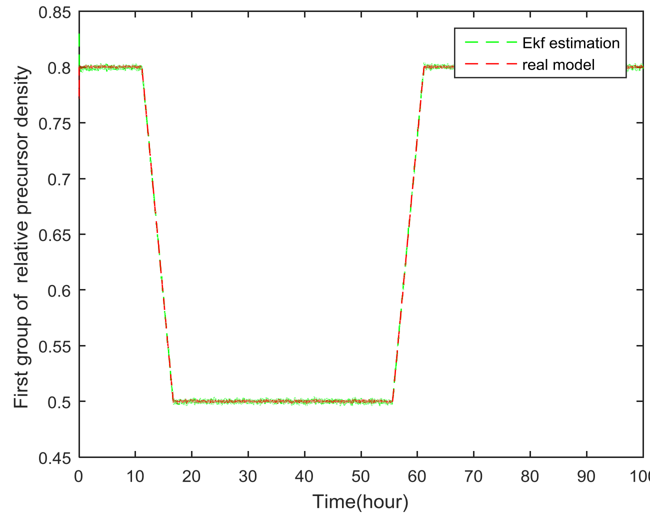

Fig. 10. First group of Relative Precursor density, C_{r1}.

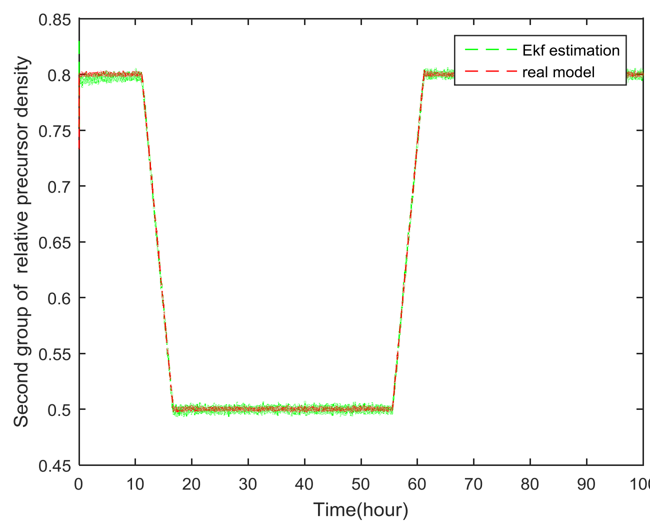

Fig. 11. Second group of Relative Precursor density, C_{r2}.

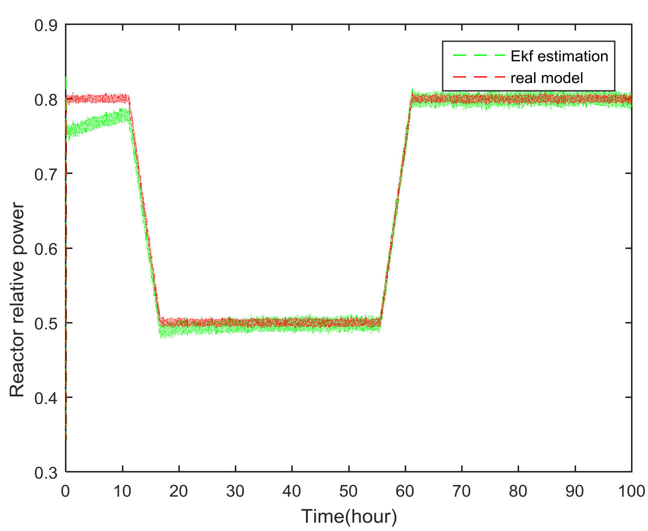

Fig. 9. Reactor relative power, n_r.

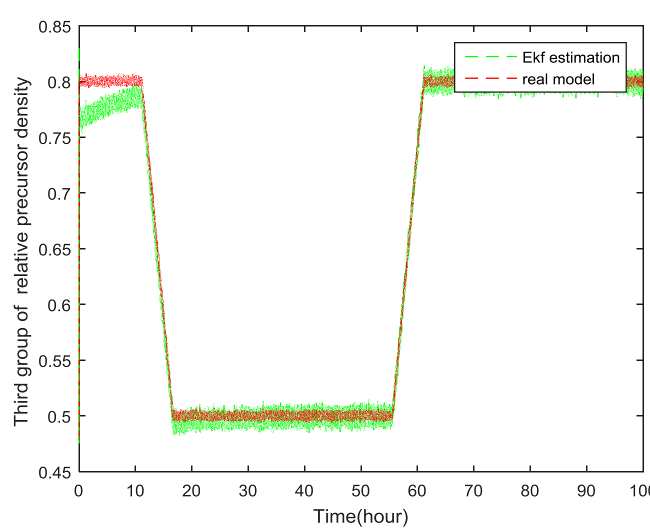

Fig. 12. Third group of Relative Precursor density, C_{r3}.

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E.K.F equations described in Section5. Real modeled and estimated relative neutron densities of reactor core have been shown in Fig. 9. Real modeled and estimated first, second and third groups of relative precursor densities of reactor core have been shown in Figs. 10–12, respectively. Real modeled and estimated iodine and xenon concentrations have been shown in Figs. 13 and 14, respectively. Real modeled and estimated Promethium and Samarium concentrations have been shown in Figs. 15 and 16, respectively. Results demonstrate that the modeled and estimated variables closely agree and the E.K.F observer follows the actual system variables in finite time accurately. Indeed, perfect tracking and good convergence to the actual states has been achieved exactly with E.K.F.

Simulation results confirm the stability and perfect tracking of Extended Kalman filter system in the presence of the parameters uncertainties and disturbance. Indeed, better tracking and convergence to the actual states for E.K.F compare to the kalman filter can be seen.

6. Comparison of the E.K.F and K.F techniques with the Luenberger observer

In this section, to clear the superiority of the designed Extend Kalman Filter (E.K.F), the comparison between the designed Luenberger observer (Ogata, 2002), Kalman filter and Extend Kalman Filter (E.K.F) has been done in the presence of the parameters uncertainties, noise and disturbances. All system parameters are perturbed by ±20% from their nominal values. Also, process and measurement noises on the reactor model have been considered.

Results show a significant improvement in the tracking of the actual system variables and an increased ability in disturbance rejection for E.K.F compare to the Luenberger observer and Kalman filter.

Fig. 17 shows the real modeled and estimated reactor relative power. Real modeled and estimated first, second and third groups

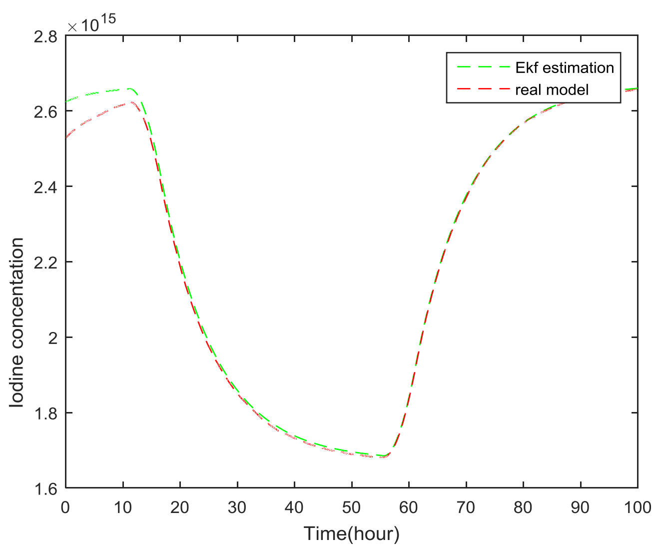

Fig. 13. Iodine concentration, *I*.

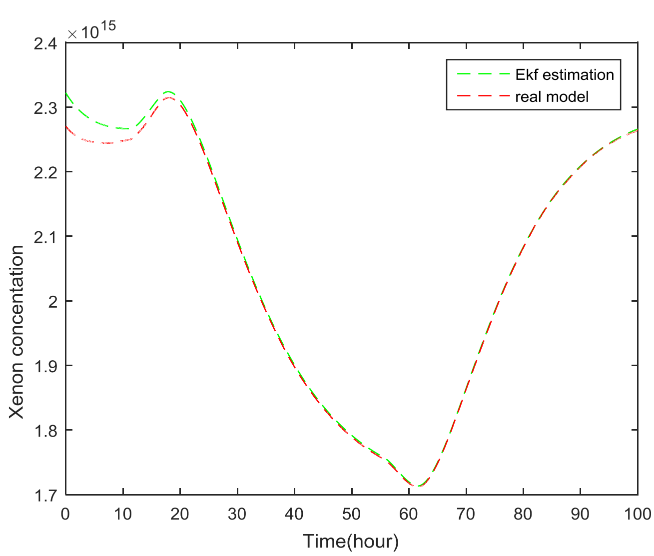

Fig. 14. Xenon concentration, *Xe*.

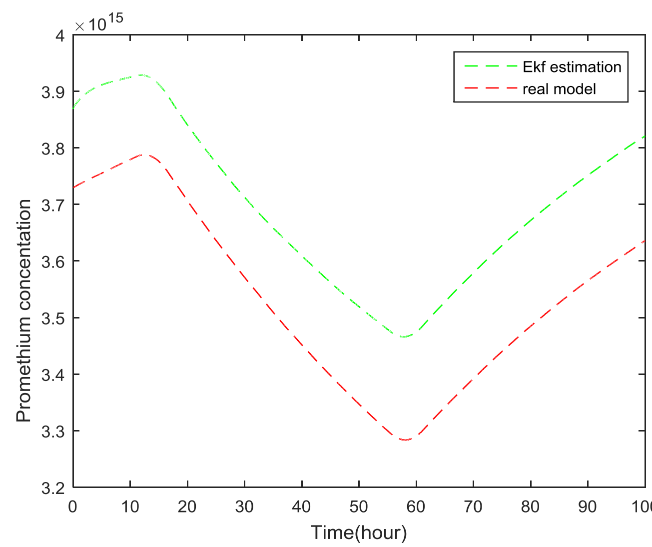

Fig. 15. Promethium concentration, Pm.

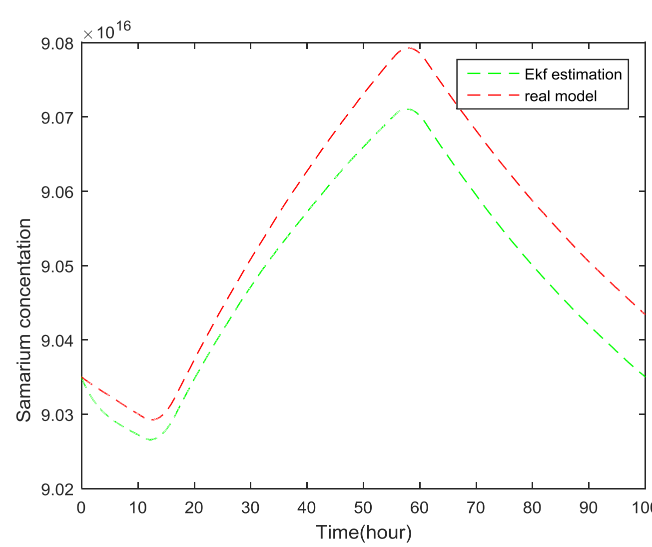

Fig. 16. Samarium concentration, Sm.

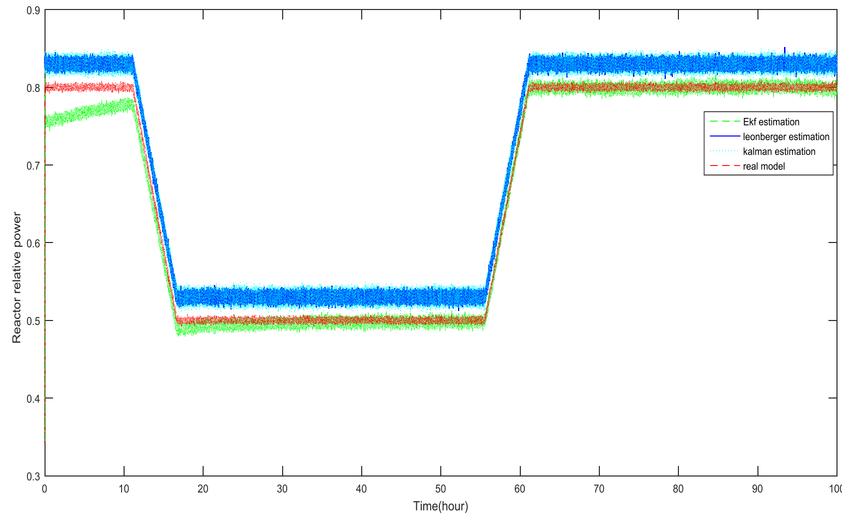

Fig. 17. Comparison of the proposed E.K.F with K.F and Luenberger observer in Reactor relative power estimation.

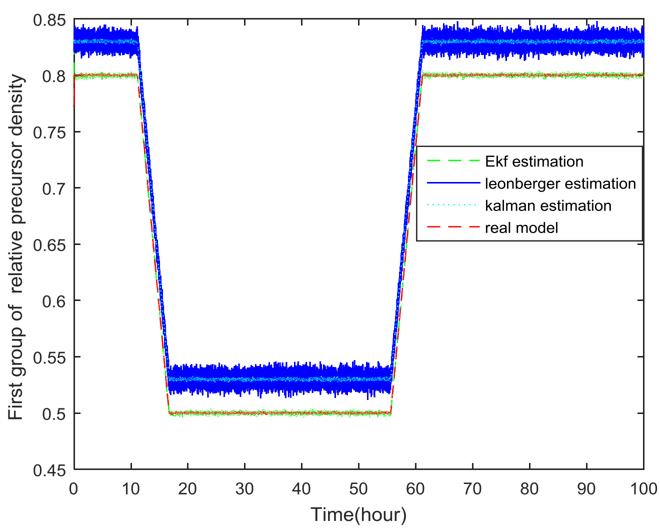

Fig. 18. Comparison of the proposed E.K.F with K.F and Luenberger observer in First group of relative precursor density estimation.

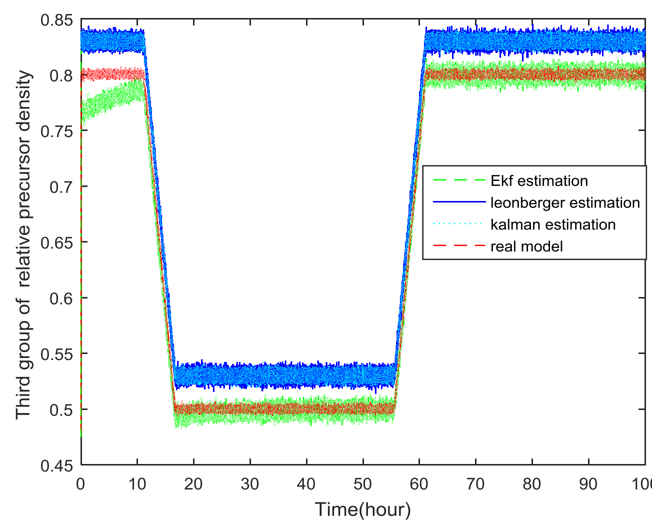

Fig. 20. Comparison of the proposed E.K.F with K.F and Luenberger observer in Third group of relative precursor density estimation.

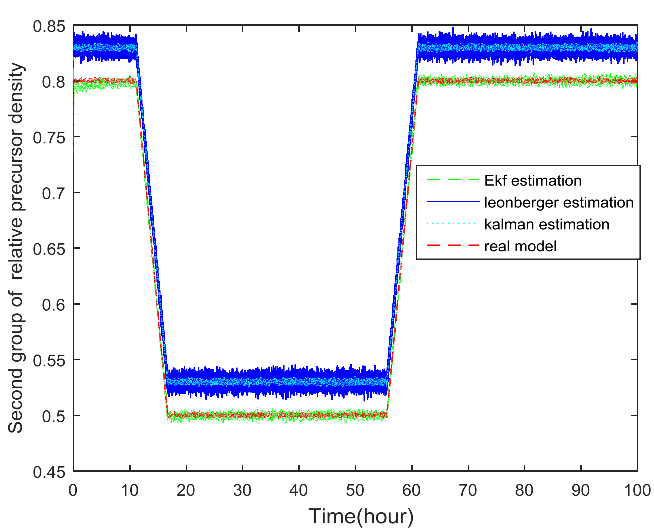

Fig. 19. Comparison of the proposed E.K.F with K.F and Luenberger observer in Second group of relative precursor density estimation.

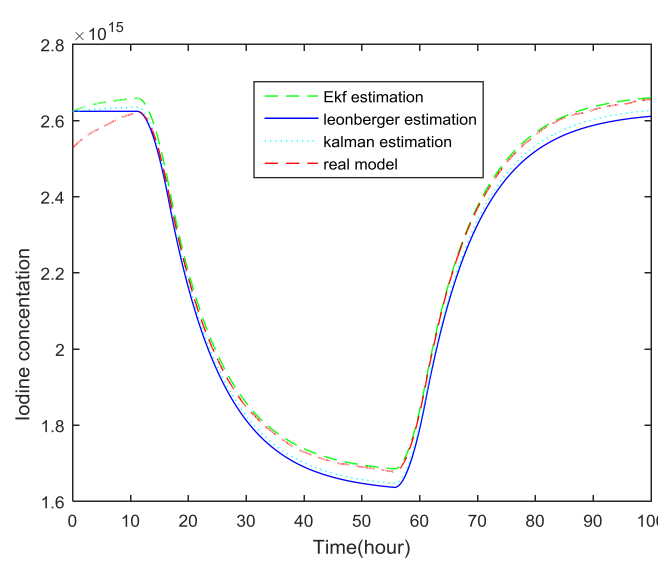

Fig. 21. Comparison of the proposed E.K.F with K.F and Luenberger observer in Iodine concentration estimation.

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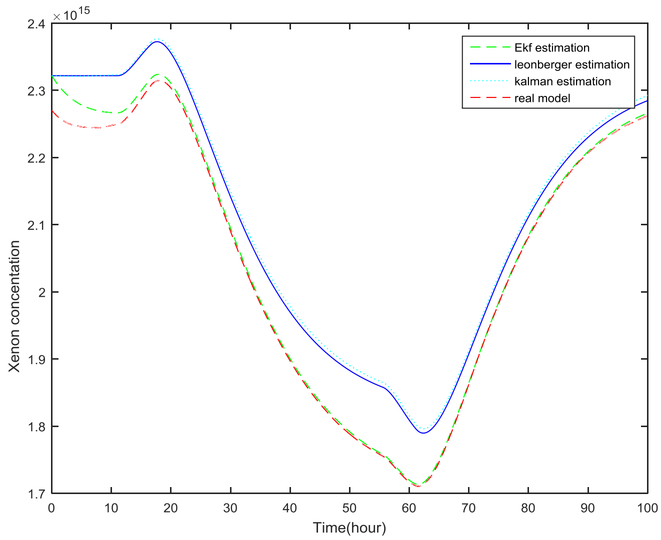

Fig. 22. Comparison of the proposed E.K.F with K.F and Luenberger observer in Xenon concentration estimation.

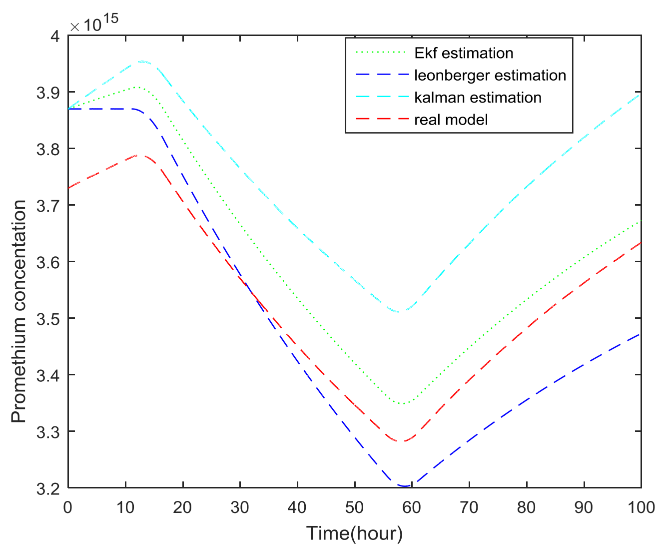

Fig. 23. Comparison of the proposed E.K.F with K.F and Luenberger observer in Promethium concentration estimation.

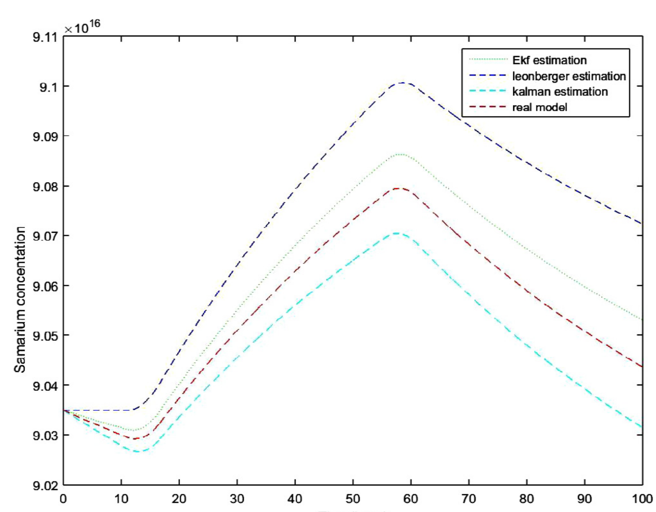

Fig. 24. Comparison of the proposed E.K.F with K.F and Luenberger observer in Samarium concentration estimation.

of relative precursor densities of reactor core have been shown in Figs. 18–20, respectively. Real modeled and estimated iodine and xenon concentrations have been shown in Figs. 21 and 22, respectively. Real modeled and estimated Promethium and Samarium concentrations have been shown in Figs. 23 and 24, respectively.

Results show that the agreement and convergence between the modeled and estimated variables is the best for the E.K.F than other methods and it is better for the Kalman filter observer compare to the Luenberger observer and proved the superiority of the designed E.K.F Observer.

7. Conclusions

- In this paper, for the first time, an Extended Kalman Filter (E.K.F) has developed for a Pressurized-Water Nuclear Reactor (PWR) using reactor power measurement to estimate the poisons concentrations and densities of delayed neutrons precursors which are difficult to measure in practice. This estimation has been done taking into account the effects of reactivity feedback due to temperature, xenon and samarium concentrations. The reactor core was simulated based on the point kinetic nuclear reactor model with three delayed neutrons groups based on the Skinner-Cohen model. This approach provides a high-performance observer on the system. At the first, a Kalman filter observer was designed based on the reactor power measurement. The results confirm the stability of Kalman filter system, but perfect tracking and good convergence to the actual states have not been achieved exactly; therefore, E.K.F system has been designed and presented to improve the performance and tracking capability of Kalman filter.

Simulation results demonstrated that the E.K.F observer followed the actual system variables accurately and was satisfactory in the presence of the parameters uncertainties and disturbance. Also, the comparison between the designed E.K.F, Kalman Filter and the Luenberger observer was done which showed a significant improvement in the actual system variables following and an increased ability in disturbance rejection for E.K.F compare to the Kalman Filter and Luenberger observer and proved the superiority of the designed Extended Kalman Filter. Also, undesirable characteristic of the sliding mode observer which is chattering did not appear in the Kalman Filter and E.K.F.

Indeed, Considering the statistical behavior in the nuclear reactors and in order to the robustness against the statistics noise of measurement and processes, application of E.K.F in nuclear reactors is very useful. Besides, in this paper, Kalman filter and E.K.F have been developed for the nonlinear systems with multi-immeasurable states such as nuclear reactors (xenon concentration, samarium concentration and delayed neutrons precursors densities) and process noise was modeled as a signal contains the disturbance on the control input and parametric uncertainties on the parameters which is the second contribution of this paper as a new proposal in the theory of Kalman filter.

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