

# Maximizing the lifetime in wireless sensor networks with multiple mobile sinks having nonzero travel times

Muhammed Emre Keskin\*, Vecihi Yiğit

Industrial Engineering Department, Engineering Faculty, Atatürk University, Erzurum, Turkey

## ARTICLE INFO

### Keywords:

Wireless sensor networks  
Mobile sinks  
Nonzero sink traveling time  
Mixed-integer linear programming

## ABSTRACT

It is a known phenomenon of the wireless sensor networks that the relay sensors, the sensors neighboring directly to the sinks, spend more energy than the rest of the sensors. This causes premature death of the relay sensors which isolates the sink(s) from the rest of the network. In order to prevent isolation of the sinks due to premature death of the relay sensors, sink mobility is offered as a remedy and that has attracted the interest of many researchers. Mobility of the sinks has been the subject of numerous studies but most of them assume that the mobile sinks travel from one point to another in negligible amount of time. However, harsh and hostile environments in which wireless sensor networks are deployed force the mobility of the sinks to be limited in real applications. Velocity of the mobile sinks for the wireless sensor networks deployed in hostile environments is so slow and consequently considerable sink traveling times occur. In this study, a mathematical model is offered that takes the sink travel times into consideration and the importance of taking sink traveling times into account is illustrated especially for the slow sinks.

## 1. Introduction

Wireless Sensor Networks (WSNs) consist of many multi-functional, tiny electronic devices called sensors which are deployed over a region of interest called sensor field. Sensors interact with each other in a wireless fashion to form a distributed platform which promotes the usage of WSNs. Each sensor senses its close proximity lying within its sensing range and the collected knowledge is transferred to either neighboring sensors or neighboring gateway nodes, called sinks. Data transfer is possible only if the receiver sensor or the sink is in the communication range of the sensor that is sensing the data. Energy capacities of the sensors are very limited and replacing the depleted batteries of the sensors is out of option due to the large number of the deployed sensors and due to the difficulty of reaching each sensor. Hence, energy loads on each sensor should be carefully planned beforehand in order to extend the WSN lifetime. WSN lifetime can be defined as the time until the first sensor death or the time until the first coverage loss in the sensor field.

It is a known fact that the sensors neighboring directly to the sinks in wireless sensor networks spend more energy than the rest of the sensors which will lead premature death of these sensors and isolation of the sink(s) from the network. This problem is named differently in several sources, such as “the crowded center effect” in [Popa, Rostamizadeh, Karp, Papadimitriou, and Stoica \(2007\)](#), as “energy hole problem” in [Li and Mohapatra \(2007\)](#) and [Wu, Chen, and Das \(2008\)](#) and “sink

neighborhood problem” in [Basagni, Carosi, Melachrinoudis, Petrioli, and Wang \(2008\)](#). Sink mobility is offered as a remedy for this phenomenon. If the sinks are taken as mobile devices, then the sensors neighboring directly with sinks changes with time as the sinks move which will distribute the extra energy load among the sensors. This will eventually help in balancing the energy loads among the sensors and extend the network lifetime. This has attracted the interest of many researchers, for instance, there are many studies seeking a trajectory for the mobile sink(s) that maximizes the network lifetime ([Basagni et al., 2008](#); [Basagni, Carosi, Petrioli, & Phillips, 2011](#); [Gatzianas & Georgiadis, 2008](#); [Papadimitriou & Georgiadis, 2005](#); [Wang, Basagni, Melachrinoudis, & Petrioli, 2005](#); [Yun & Xia, 2010](#)). Most of the wireless sensor network studies with mobile sinks consider that sinks travel from one point to another in negligible amount of time. There are very few studies that takes the sinks as energy limited devices. For instance, [Liang, Luo, and Xu \(2010\)](#) consider the sink as a mechanically driven device using petrol or electricity implying that the sink has to visit the petrol station or an electricity hop to renew its battery periodically. The authors not only propose a mathematical model that is similar to the one given in [Basagni et al. \(2008\)](#) but also add upper limit on the total travel lengths and on the distance between two successive sojourn points of the sinks in order to reflect the limited nature of the sinks. Alternatively, [Keskin, Altunel, Aras, and Ersoy \(2011\)](#) propose two

\* Corresponding author.

E-mail address: [m.emre.keskin@gmail.com](mailto:m.emre.keskin@gmail.com) (M.E. Keskin).

mathematical models in which sink travel times are not taken as zero implying that the nonzero traveling times and the data accumulated during the sink travel times are taken into consideration. However, they assume a single mobile sink which travels on a constant tour numerous times. As far as we know, all the studies in the literature employing multiple mobile sinks assume that sinks are energy limitless devices and the travel times of the sinks are negligible. Nevertheless, mobility of the sinks for the wireless sensor networks deployed in hostile environments, i.e., the marine wireless sensor networks, should be limited. Velocity of the mobile sinks for marine wireless sensor networks is relatively slow implying considerable sink traveling times which should be taken into consideration during the determination of the routes for the sinks. We propose a mathematical model considering sink travel times and we show the importance of taking sink traveling times into consideration for multiple mobile sinks. Another point that comes to mind is about the relocation costs of the sinks. However, sinks travel from one point to another only between the periods and they stand on their locations within the periods implying that each sink travels a limited distance. Therefore, the relocation cost that occurs during sink travels is also negligible compared to the revenue gained by having a network with longer lifetime.

This paper contributes to the literature in 3 main ways:

1. A mathematical model that takes the sink travel times for multiple mobile sinks is provided.
2. It is shown that considering the sink travel time is important under certain conditions. We illustrated the conditions for which consideration of the sink travel times is relevant/irrelevant.
3. We developed an optimization framework that WSN applications can follow especially for the cases where the sinks' mobility is limited.

Organization of the paper is as follows. We provide a brief review of the literature in the next section. We provide the well-known mathematical model of the literature which does not take the sink traveling times into account and our new model that incorporates nonzero sink traveling times in the Mathematical Models section. In the Test Bed section, we talk about the determination process of the parameter values and in Numerical Results section, we prove that the sink travel times should be taken into account for the extreme cases, i.e., for the marine WSNs wherein sinks travel in relatively slow velocities. Finally, we point out future research directions and conclude in the final section.

## 2. Literature review

Mobility in WSNs has been studied extensively. Here, we specifically focus on mobile WSN studies including a mathematical model. Gu, Bozdag, Ekici, Ozguner, and Lee (2005,) try to find a path, which is to be repeated periodically, that ensures zero data loss. Somasundara, Ramamoorthy, and Srivastava (2007) also analyze the same problem but consider the problem as a single machine scheduling problem instance by taking sensors as jobs while the sink is considered as the machine. Alternatively, Vincze, Fodor, Vida, and Vidács (2006) respectively assign positive and negative charges to the sinks and sensors depending on the residual energies of the sensors and employ the Coulomb rule in a repetitive manner until the locations of the sinks converge. On the contrary, Nesamony, Vairamuthu, Orłowska, and Sadiq (2006) clusters the sensors into groups and the sink is forced to visit at least one sensor from each group implying a traveling salesman problem with neighborhoods. Additionally, Wang et al. (2005) provide a linear program to find the sink visit points and sojourn times of the sinks on the points. Basagni et al. (2008) extend the setting of Wang et al. (2005) in order to provide the order of the sinks together with the sojourn times. Conversely, the model of Wang et al. (2005) is extended by adding multiple mobile sinks in Basagni et al. (2011). On the other hand, Gandham, Dawande, Prakash, and Venkatesan (2003) develop the model of Wang et al. (2005) so that the data flows on each arc

**Table 1**

Summary of the depth of integration of the literature studies. (CP: Coverage Problem, ASP: Activity Scheduling Problem, SLP: Sink Location Problem, SRP: Sink Routing Problem, MS: Multiple Sinks)

Papers	CP	ASP	SLP	SRP	DRP	MS
Popa et al. (2007)				X		
Li and Mohapatra (2007)				X		
Wu et al. (2008)				X		
Gu et al. (2005,)				X		
Somasundara et al. (2007)				X		X
Vincze et al. (2006)				X		X
Nesamony et al. (2006)				X		
Wang et al. (2005)				X		
Basagni et al. (2008)				X		
Basagni et al. (2011)				X		X
Liang et al. (2010)				X		
Keskin et al. (2011)				X	X	
Gandham et al. (2003)				X	X	X
Azad and Chockalingam (2006)				X	X	X
Alsalihi et al. (2007)				X	X	X
Jun and Hubaux (2007)				X	X	
Papadimitriou and Georgiadis (2005)				X	X	
Gatzianas and Georgiadis (2008)				X	X	
Yun and Xia (2010)				X	X	
Yun et al. (2013)				X	X	
Behdani et al. (2012)				X	X	
Luo and Hubaux (2010)				X	X	X
Güney, Aras, Altınel, and Ersoy (2010)			X		X	X
Güney, Aras, Altınel, and Ersoy (2012)		X	X		X	X
Türkoğulları, Aras, and Altınel (2009)		X	X		X	X
Türkoğulları, Aras, Altınel, and Ersoy (2010b)		X	X	X	X	X
Türkoğulları, Aras, Altınel, and Ersoy (2010a)		X	X	X	X	X
Keskin (2017)		X	X		X	X
Guimarães et al. (2020)			X		X	X
This Study				X	X	X

of the network are also decision variables to be optimally determined for the maximum network lifetime. Azad and Chockalingam (2006) are also similar to the study of Alsalihi, Akl, and Hassanein (2007) but concentrate on minimizing the maximum energy used by the sensors. Alsalihi et al. (2007) also borrow the same framework but try to maximize the minimum residual energy of the sensors at the end of the network lifetime. A very recent study by Guimarães, Frigieri, and Sakai (2020) also adopts an equivalent strategy in which sinks are also responsible from recharging sensors' energies implying that the problem of energy hole is *automatically* handled by making the sinks to be subjected to communication activity control. These studies present mathematical modeling approaches but focus on energy issues rather than direct maximization of the lifetime. On the contrary, objective of Jun and Hubaux (2007) is to maximize the network lifetime while a single mobile sink that moves in a circular network area periodically. Similarly, Papadimitriou and Georgiadis (2005) try to find the maximum network lifetime under data flow balance constraints on a grid network area. A distributed solution strategy is developed in Gatzianas and Georgiadis (2008) for a model that is similar to the model of Papadimitriou and Georgiadis (2005). Yun and Xia (2010) also extend the model of Papadimitriou and Georgiadis (2005) so that the model becomes tolerable for delay tolerances. Yun, Xia, Behdani, and Smith (2013) and Behdani, Yun, Smith, and Xia (2012) develop distributed solution strategies for the model of Yun and Xia (2010). Finally, Luo and Hubaux (2010) seek for the maximum network lifetime under the presence of multiple mobile sinks. However, sinks are assumed to travel instantaneously in zero time from one point to another.

None of the works mentioned above deeply integrates the coverage problem, sink location or sink routing problem and data routing problem. There are a few studies which integrate more than one problem simultaneously. For instance, in Güney et al. (2010) authors try to find the optimal sink locations and data routes. Güney et al. (2012) integrate coverage problem with sink location and data routing problem using the results of Güney et al. (2010). The setting of Türkoğulları

et al. (2009) is also similar but the authors also integrate the sensor scheduling problem. Moreover, Türkoğulları et al. (2010b) work on an extended setting in which sink placement is also a concern on top of sensor placement, activity schedules of the sensors and data routes. Besides, a column generation based heuristic is derived in Türkoğulları et al. (2010a) for the model given in Türkoğulları et al. (2010b). Finally, Keskin (2017) integrates the coverage, sink routing, and activity scheduling problems with data routing problem for multiple mobile sinks.

In Table 1, a summary of the reviewed studies with respect to the depth of integration they have is provided. First column of the table includes the reference numbers of the papers and the following four columns stand for the coverage, activity scheduling, sink location and sink routing problems. The last column shows whether the paper considers Multiple Sinks (MS) or not. The last line is written for this study. A final comment is that all the studies listed in Table 1 but one do not consider sink travel time into consideration. The only study that takes sink travel time into account is due to Keskin et al. (2011), but they assume that there is a single mobile sink. In this study, we extend the study of Keskin et al. (2011) by considering sink travel times for multiple mobile sinks and fill the gap existing in the literature.

### 3. Mathematical formulations

In this section, we first give the mathematical model adopted in Luo and Hubaux (2010) which represents the state of the art model that does not take the sink travel times into consideration for multiple mobile sinks. Later on, we extend the model to a new one in which the sink travel times and the data accumulated during the sink travel times are taken into account. The models are respectively called as the Basic Model and the Extended Model. We first provide the definitions of sets and parameters, and the decision variables used in the models respectively in Table 2 and in Table 3.

Now, we give the formulations of the models in the following.

Basic Model:

$$\text{maximize } \sum_{i \in \mathcal{T}} w_i \quad (1)$$

subject to:

$$\sum_{j: i \in I_j} x_{jit} + hw_t = \sum_{j \in I_i} x_{ijt} + \sum_{l \in L_i} y_{ilt} \quad i \in I, t \in \mathcal{T} \quad (2)$$

$$\sum_{i \in \mathcal{T}} \left( c^r \sum_{j: i \in I_j} x_{jit} + c^s hw_t + \sum_{j \in I_i} c'_{ij} x_{ijt} + \sum_{l \in L_i} c'_{il} y_{ilt} \right) \leq E \quad i \in I \quad (3)$$

$$\sum_{i: l \in L_i} y_{ilt} \leq M \sum_{p \in \mathcal{P}} z_{plt} \quad l \in L, t \in \mathcal{T} \quad (4)$$

$$\sum_{i \in L} z_{plt} = 1 \quad p \in \mathcal{P}, t \in \mathcal{T} \quad (5)$$

$$z_{plt} \in \{0, 1\} \quad p \in \mathcal{P}, l \in L, t \in \mathcal{T} \quad (6)$$

$$w_i, x_{ijt}, y_{ilt} \geq 0 \quad i, j \in I, l \in L, t \in \mathcal{T} \quad (7)$$

In the objective function (1), network lifetime, which is defined as the summation of the period lengths, is maximized. It should be noted that a period is defined as the time that passes between two consecutive sink configurations. For instance, if the model decides to spend 0.5 h for a particular sink location scheme before relocating the sinks, the related period length will be 0.5 h. Constraint (2) forces the summation of the total data coming from the neighboring sensors and the data produced by the sensor is equal to the total data sent from the sensor to neighboring sensors and the sinks located at neighboring locations for each period and for each sensor. Note that in the first term, we sum  $x_{jit}$  values over  $j : i \in I_j$  implying that summation includes only  $x_{jit}$  values for sensor(s)  $j$  for which sensor  $i$  lays within their communication range. If a sensor  $j$  is far away from sensor  $i$ , then it cannot directly reach to sensor  $i$  implying that  $i \notin I_j$  and in this case,  $x_{jit}$  value is null.

This is why we exclude the  $x_{jit}$  values for which  $j : i \notin I_j$  from the first term. Thus, we sum  $x_{jit}$  values which are possibly nonzero only for the sensors that are near sensor  $i$ , so that the summation is over  $j : i \in I_j$ . As a result, constraint (2) ensures the data flow balance throughout the network lifetime for each sensor. Constraint (3) makes it sure that the total energy spent by each sensor for data receiving, sensing and processing, and data transfer throughout the network lifetime is less than or equal to the initial battery energy of the sensors. Constraint (4) is written for each period and it avoids transfer of data to the sink locations wherein no sink is placed. Constraint (5) states that each of the sinks has to be placed at one of the sink locations at each period. Finally, constraint (6) puts usual binary restrictions on the sink location variables while constraint (7) indicates usual nonnegativity restrictions.

The basic model of Luo and Hubaux (2010) assumes that each sink is allowed to go from one point to another between periods and that the time for the relocation takes negligible amount of time. This assumption is usually valid for most of the WSN environments for which sink relocations are handled easily within short amount of times. On the other hand, if the WSN is deployed within a hostile environment such as a military WSN for observation of enemy activities, sink relocations are either impossible or can be made in very long times. Similarly, sink relocations can be very difficult and can take very long amount of times due to environmental conditions for WSNs deployed on the ocean floors, i.e., submarine WSNs. Therefore, there is a need for a new mathematical modeling approach in which sink relocation times are considered in the network lifetime and the data accumulated during sink relocations are taken into consideration. We fulfill this need by the Extended Model given in the following.

Extended Model:

$$\text{maximize } \sum_{i \in \mathcal{T}} w_i + a_t \quad (8)$$

subject to:

(4)–(7)

$$\sum_{j: i \in I_j} x_{jit} + h(w_t + a_t) = \sum_{j \in I_i} x_{ijt} + \sum_{l \in L_i} y_{ilt} \quad i \in I, t \in \mathcal{T} \quad (9)$$

$$\sum_{i \in \mathcal{T}} \left( c^r \sum_{j: i \in I_j} x_{jit} + c^s h(w_t + a_t) + \sum_{j \in I_i} c'_{ij} x_{ijt} + \sum_{l \in L_i} c'_{il} y_{ilt} \right) \leq E \quad i \in I \quad (10)$$

$$q_{plmt} = z_{pl(t-1)} z_{pmt} \quad p \in \mathcal{P}, l, m \in L, t \in \mathcal{T} / \{1\} \quad (11)$$

$$a_t \geq q_{plmt} \frac{d_{lm}}{v} \quad p \in \mathcal{P}, l, m \in L, t \in \mathcal{T} / \{1\} \quad (12)$$

$$q_{plmt} \geq 0 \quad p \in \mathcal{P}, l, m \in L, t \in \mathcal{T} / \{1\} \quad (13)$$

This time, lifetime of the network is defined not only the summation of the period lengths but also the summation of the relocation times between the periods. Constraints (4)–(7) of the basic model is still valid for the extended model, so we do not rewrite them but refer to their numbers. Constraints (2) and (3) of the basic model transform to constraints (9) and (10) in the extended model. Note that the data accumulated during relocation times between periods is taken into consideration in the data flow balance constraints and the energy constraints as well. In constraint (11) we keep track of the routes of the sinks for each sink and for each period and we make the passing times to be at least the largest travel time of the sinks by the help of constraint (12). Finally, nonnegativity restrictions on the new variable  $q_{plmt}$  are put by the help of constraint (13).

Although the extended model takes the sink travel times into consideration, there is a problematic issue in the formulation. Constraint (11) defines  $q_{plmt}$  as the product of  $z_{pl(t-1)}$  and  $z_{pmt}$  and multiplication makes the model nonlinear. Therefore, instead of constraint (11) including the nonlinear product term, we define the following three constraints which do the same job with constraint (11) without harming the

**Table 2**  
Definition of sets and parameters used in the models.

Sets and Parameters	Definitions
$\mathcal{T}$	Set of time periods
$\mathcal{I}$	Set of sensors
$\mathcal{L}$	Set of sink locations
$\mathcal{P}$	Set of actual sinks
$v$	Velocity of the sinks
$\mathcal{I}_i$	Set of neighboring sensors of sensor $i$
$\mathcal{L}_i$	Set of neighboring sink locations of sensor $i$
$h$	Data produced by each sensor per unit time
$P$	Number of sinks to be deployed at each period
$E$	Initial battery energy of the sensors
$c^r$	Energy spent by the sensor for each bit of data it receives per unit time
$c^s$	Energy spent by the sensor for sensing and processing data per unit time
$c_{ij}^t$	Energy spent by sensor $i$ for sending unit data to neighboring sensor $j$ or to a sink located at $j$
$d_{lm}$	Distance between sink visit locations $l$ and $m$

**Table 3**  
Definition of variables used in the models.

Variables	Definitions
$w_t$	Length of period $t$
$a_t$	Duration between periods $t - 1$ and $t$
$z_{pl t}$	Indicates whether or not sink $p$ is located at $l$ at period $t$
$x_{ijt}$	Amount of data sent by sensor $i$ to neighboring sensor $j$ at period $t$
$y_{ilt}$	Amount of data sent by sensor $i$ to the sink located at point $l$ at period $t$
$q_{pl m t}$	Indicates whether or not sink $p$ is located at point $l$ at period $t - 1$ and moved to point $m$ at period $t$

linearity of the model:

$$q_{pl m t} \leq z_{pl(t-1)} \quad p \in \mathcal{P}, l, m \in \mathcal{L}, t \in \mathcal{T} \setminus \{1\} \quad (14)$$

$$q_{pl m t} \leq z_{p m t} \quad p \in \mathcal{P}, l, m \in \mathcal{L}, t \in \mathcal{T} \setminus \{1\} \quad (15)$$

$$q_{pl m t} \geq z_{pl(t-1)} + z_{p m t} - 1 \quad p \in \mathcal{P}, l, m \in \mathcal{L}, t \in \mathcal{T} \setminus \{1\} \quad (16)$$

Note that if one of the  $z_{pl(t-1)}$  and  $z_{p m t}$  are 0, then  $q_{pl m t}$  is set to zero by the collaborative effort of constraints (13) and (14) or (13) and (15) depending on which one (or both) of  $z_{pl(t-1)}$  and  $z_{p m t}$  is zero. On the other hand, if  $z_{pl(t-1)}$  and  $z_{p m t}$  are both 1, then  $q_{pl m t}$  is set to 1 by constraints (14) and (16). Hence, we obtain the product of  $z_{pl(t-1)}$  and  $z_{p m t}$  in a linear fashion.

A final discussion can be made about the complexity and practicality of the proposed mathematical optimization method for the WSN design. It is a known phenomenon that general mixed integer programming is known to be an NP-complete problem implying that we must resort to heuristic procedures for the solution of the moderate and large sized problem instances. Therefore, we can solve only small sized problem instances by depending solely on the commercial mixed integer programming solvers like Gurobi or Cplex. This implies that if one aims to design a large sized WSN by the help of the proposed extended model, she has to come up with a heuristic solution strategy.

#### 4. Test bed

We generate five different instances having 40, 60, 80, 100 and 150 sensors. Locations of the sensors are chosen such that they form a grid. Horizontal and vertical distances between neighboring sensors are taken as 15 m. Grid is constructed in such a way that side lengths of the grid are integers and as close as possible to each other. For instance, size of the grids are respectively  $5 \times 8$  and  $10 \times 10$  for the cases with 40 and 100 sensors. On the other hand, number of sink visit locations is assumed to be the half of the number of sensors and visit locations of the sinks also possess a grid structure that is nested with the grid of the sensors. The four outmost corner points of the sink grid is chosen as the center points of the four outmost corner cell of the sensor grid. As a consequence of constructing sink grid in that manner, horizontal and vertical distances between neighboring sink visit locations are respectively  $15 \times (n_1 - 2) / (m_1 - 1)$  and  $15 \times (n_2 - 2) / (m_2 - 1)$  where sensor grid is of size  $n_1 \times n_2$  and sink grid is of size  $m_1 \times m_2$ . We give

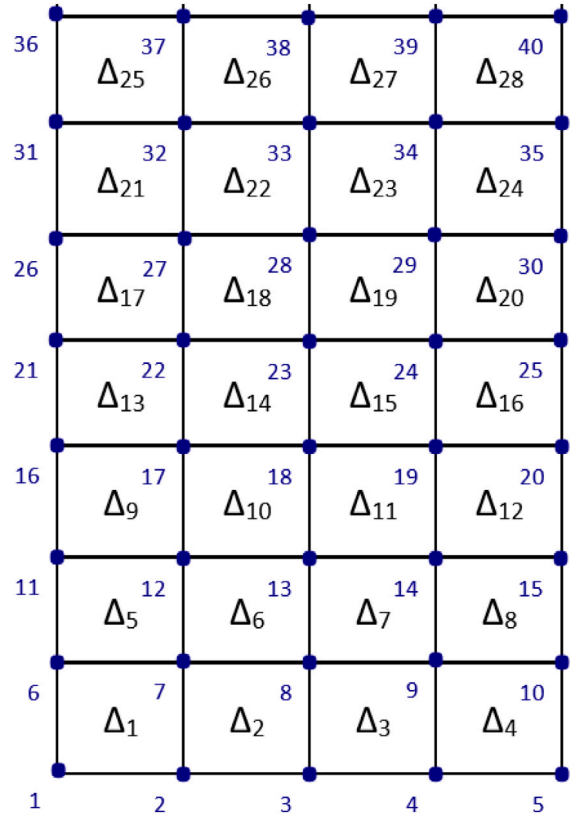


Fig. 1. Sensor area for 40 sensors.

the sensor area for 40 number of sensors in Fig. 1. Note that the sensors are illustrated by dots while sink visit points are shown by triangles.

It should be noted that the sensor area possesses a compact and regular shape when a grid structure is assumed. A randomly generated topology with the same number of sensors would be much more spread compared to the grid topology. Hence, the sinks would probably travel

much more for WSNs with random topologies. This implies that consideration of sink travel times would contribute to the network lifetime much more for WSNs with random topology. Therefore, if we achieve to illustrate the importance of taking sink travel times into consideration for the WSNs with grid structures in which sink mobilities are relatively limited due to the compact shape of the topology, that conclusion can easily be extended for the WSNs with random topologies. This is the main reason of our conservative approach for selecting grid topology over random topology.

Moreover, the number of actual sinks that are to be placed on the sink visit locations at each period is taken to be 3, i.e.,  $P = 3$ . We work with 9 different sink velocities which are 0.1, 0.5, 1, 2, 5, 10, 20, 50, 100, and all velocities are in meter/hour units. We assume that sensors have 22 meters sensing range and 80 meters communicating range. Moreover, sensors are assumed to have 20000 J battery energies and amount of data produced by each sensor per hour is assumed to be 4096 bits, i.e.,  $h = 4096$  bits/hour. Energy spent by each sensor for sensing and processing the data per hour is taken as  $c^s = 50$  nJ while the energy spent by each sensor for receiving a bit data is equal to  $c^r = 50$   $\mu$ J. Finally, the energy spent by sensor  $i$  for transferring a bit data to neighboring sensor  $j$  is equal to  $c_{ij}^t = \delta_1 + \delta_2 d_{ij}^2$  where  $\delta_1$  and  $\delta_2$  are respectively taken as 50  $\mu$ J per bit and 100 nJ per bit  $\times m^2$  while  $d_{ij}$  is the distance between sensors  $i$  and  $j$ . We have referred to Keskin et al. (2011) to determine the parameter values.

## 5. Numerical results

The models are coded in Visual Studio Environment in C# programming language. We refer to the state-of-the-art mixed integer programming solver Gurobi for the solution of the mathematical models. We let Gurobi to run for at most 3 h for each of the instances. If the solver finds the optimal solution before the allotted computation time, then it immediately reports the optimal solution and halts running for the instance. If the optimal solution is not found within the computation time, then it reports the best feasible solution it finds after the computation time finishes.

For the purpose of verification of the extended model we have to indicate that it is possible to generate longer network lifetimes by consideration of the nonzero sink travel times. We coded the basic model for the parameter values defined in the previous section to obtain the network lifetimes without considering nonzero sink traveling times. These network lifetimes are indeed inflated since sinks do not travel instantaneously in zero amount of times in reality while they are assumed to do so. In order to find the real network lifetimes corresponding to the sink relocation schemes found by the basic model, values of the  $z$  variables, which represent the movements of the sinks in the models, are fixed in the extended model by the ones found by the basic model. In other words, once we find the values of the  $z$  variables from the basic model we record their values and we fix the values of the  $z$  variables with the recorded values in the extended model. The model obtained by fixing the  $z$  variables of the extended model by the ones found by the basic model is called as the *fixed model* in the sequel. Solution of the fixed model indicate the real network lifetimes corresponding to the sink relocation schemes found by the basic model.

We tabulate the lifetimes (in hours) found by the basic model and the lifetimes found by solving the fixed model in Table 4.

It can be seen from Table 4 that objective function values found by the basic model are very high compared to those found by the fixed model implying that one who does not take the sink velocities into consideration may mistakenly think that she will have long network lifetimes while their networks will last much shorter. If the sink velocities are too low, i.e.,  $v = 0.1$  meters per hour, real network lifetimes even drops to 0 for networks with 60, 80, 100 and 150 sensors meaning that sinks are too slow so that the network cannot live for the travel time of the sinks. It is also possible to observe that as the sinks get faster, i.e., the sink velocities become higher, than the

difference between the network lifetimes found by the basic model and the fixed model gets narrower meaning that neglecting the nonzero sink travel times harms less if the sinks are faster. Consequently, it is obvious that one should consider the nonzero sink travel times while determining the sink relocation schemes especially for the applications with harsh environments that naturally limits the sink velocities. For better visualization, we represent the data given in Table 4 in Fig. 2 below. BM stands for the results of the basic model and the rest of the figure includes the real lifetime of the results found by basic model for varying sink speeds. Note that horizontal axis of Fig. 2 is not scaled, for instance, the distance between 0.1 and 0.5 is equal to the distance between 50 and 100. However, we deliberately formed the figure in that way in order not to miss out the facts that the lifetime is negatively affected from not considering sink travel times for very slow sinks, and not harmed at all for faster sink speeds. The precision between 0.1 and 0.5 is especially important and not scaled to keep it visible in the figure.

In addition, we report the network lifetimes found by the extended model in Table 5. One can extract the fact from Table 5 that it is possible to extend the network lifetimes by considering the sink travel times especially for the slower sink velocities. For instance, if the sink velocity  $v$  is 0.1 meters per hour, it is possible to obtain nonzero network lifetimes by considering sink travel times. One may also observe that as the sinks get faster, network lifetimes found by the extended model converge to the ones found by the basic model as the harm due to neglecting sink travel times gets less and less for fast sink velocities. Therefore, one may neglect the nonzero sink travel times and assume that sinks have infinite velocities that jump from one point to another instantaneously if the sink velocities are high enough, i.e.,  $v$  is larger than or equal to around 100 meters per hour. However, if the sinks are slower, as in the case for the WSNs deployed in deep seas, then one has to take the nonzero sink travel times into consideration in order to obtain longer network lifetimes. We also summarize the data of Table 5 in Fig. 3 below.

Finally, in Tables 6 and 7, we give the maximum and average travel lengths of the sinks in meters for both the basic model and the extended model with varying sink speeds. A counter intuitive observation is that considering sink travel time does not reduce the travel lengths. Positive impact of sink mobility on the network lifetime has been shown many times empirically in the literature. Hence, consideration of sink travel time does not motivate the sinks to travel less since the network lifetime would have been affected negatively, otherwise. On the contrary, sinks continue to travel for the extended model scenarios as long as they do for the basic model scenario, but the travel schemes are determined so that the network lifetime is not harmed much from the consideration of the travel lengths of the mobile sinks.

## 6. Conclusions and discussions

In this study, we analyze the effect of considering nonzero sink traveling times especially for the extreme cases like the ones for submarine WSNs wherein the sinks have to move very slowly due to the hostile environmental conditions. We offer two mathematical models. The first one, which is called as the basic model, is from the literature representing the situations in which sinks are fast enough so that neglecting the sink traveling times does not harm the solution quality. We develop the second model to represent the hostile situations in which consideration of the sink traveling times is important. We are able to show with extensive numerical experiments that considering the nonzero sink travel times in the network lifetime and the data accumulated during the sink travel times is important and it is possible to extend the network lifetime considerably by taking the nonzero sink travel times into account especially when the sink velocities are relatively low.

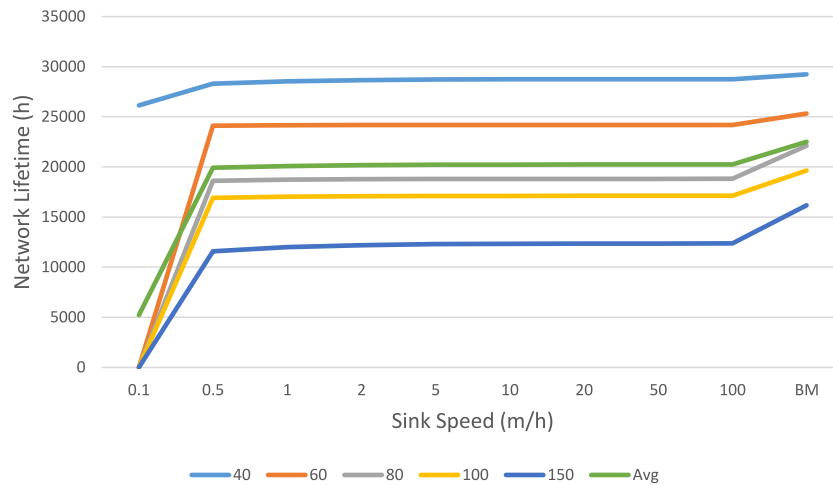
This work can be extended in several ways. First of all, an interesting extension of the models with finite storage capacities of the sensors would be developed. This will generalize the proposed extended

**Table 4**  
Lifetimes (in hours) found by the basic model and their real counterparts.

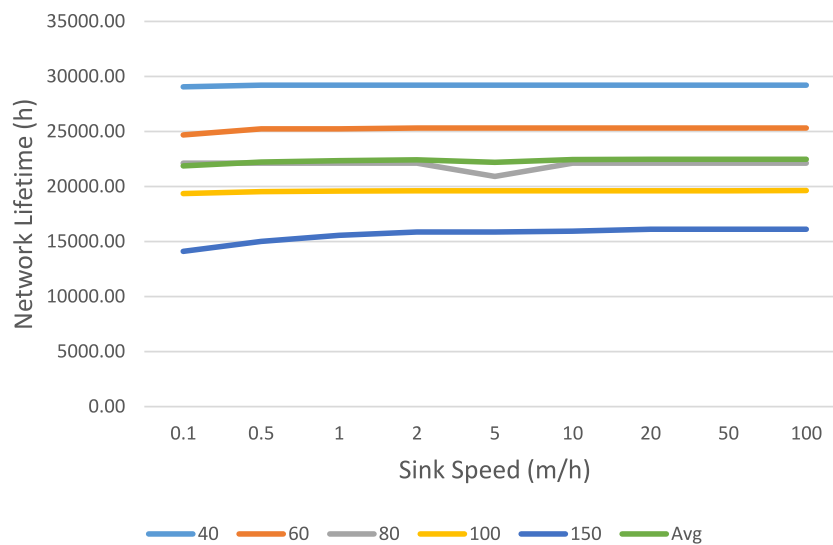
n	Basic model	Fixed model								
		$v = 0.1$	$v = 0.5$	$v = 1$	$v = 2$	$v = 5$	$v = 10$	$v = 20$	$v = 50$	$v = 100$
40	29238.9	26143.29	28314.39	28545.71	28657.7	28721.29	28741.44	28751.39	28757.33	28759.31
60	25323.03	0	24121.6	24165.27	24173.05	24175.77	24176.55	24176.94	24177.17	24177.25
80	22121.29	0	18627.45	18726.72	18772	18798.5	18807.25	18811.62	18814.23	18815.1
100	19644.9	0	16923.22	17027.95	17076.13	17104.14	17113.28	17117.82	17120.55	17121.45
150	16162.85	0	11582.72	12011.89	12198.73	12303.25	12336.77	12353.19	12362.94	12366.17
Avg	22498.19	5228.66	19913.88	20095.51	20175.52	20220.59	20235.06	20242.19	20246.44	20247.86

**Table 5**  
Lifetimes found by the extended model.

n	Extended model								
	$v = 0.1$	$v = 0.5$	$v = 1$	$v = 2$	$v = 5$	$v = 10$	$v = 20$	$v = 50$	$v = 100$
40	29052.36	29207.25	29207.25	29207.25	29207.25	29207.25	29207.25	29207.25	29207.25
60	24680.24	25236.84	25236.84	25308.76	25323.03	25323.03	25323.03	25323.03	25323.03
80	22119.12	22119.12	22119.12	22119.12	20921.83	22119.12	22121.29	22121.29	22121.29
100	19366.27	19529.46	19586.84	19611.83	19611.83	19611.83	19611.83	19611.83	19644.9
150	14112.65	14999.98	15571.71	15874.25	15874.25	15945.62	16114.41	16114.41	16114.41
Avg	21866.13	22218.53	22344.35	22424.24	22187.64	22441.37	22475.56	22475.56	22482.18



**Fig. 2.** Basic model results and their real counterparts.



**Fig. 3.** Extended model lifetimes.

model and prevent long delays of the collected data at the storage areas of the sensors because of long stays until the sink arrives. Secondly,

developing a heuristic solution strategy for solving large instances is another option.

**Table 6**  
Maximum travel lengths of the sinks.

n	Basic model	Extended model								
		v = 0.1	v = 0.5	v = 1	v = 2	v = 5	v = 10	v = 20	v = 50	v = 100
40	113.28	127.50	72.04	72.04	72.04	72.04	72.04	72.04	72.04	72.04
60	132.14	158.45	130.58	130.58	194.30	132.14	132.14	132.14	132.14	132.14
80	169.29	169.29	169.29	169.29	169.29	169.29	169.29	169.29	169.29	169.29
100	189.69	189.69	189.69	189.69	189.69	189.69	189.69	189.69	189.69	189.69
150	189.20	303.32	179.52	216.62	196.20	196.20	209.61	189.20	189.20	189.20

**Table 7**  
Average travel lengths of the sinks.

n	Basic model	Extended model								
		v = 0.1	v = 0.5	v = 1	v = 2	v = 5	v = 10	v = 20	v = 50	v = 100
40	102.18	98.58	63.03	63.03	63.03	63.03	63.03	63.03	63.03	63.03
60	84.85	118.01	127.02	127.02	105.57	84.85	84.85	84.85	84.85	84.85
80	157.15	150.46	150.46	150.46	150.46	150.46	150.46	157.15	157.15	157.15
100	128.83	122.04	174.37	132.82	132.82	132.12	132.12	132.12	132.12	128.83
150	134.44	202.31	161.04	168.14	159.00	159.00	158.62	134.44	134.44	134.44

## CRediT authorship contribution statement

**Muhammed Emre Keskin:** Idea of the problem, Mathematical Modeling, Coding. **Vecihi Yiğit:** Literature review, Exposition of the results, Writing- review & editing.

## References

- Alsalihi, W., Akl, S., & Hassanein, H. (2007). Placement of multiple mobile base stations in wireless sensor networks. In *IEEE International symposium on signal processing and information technology* (pp. 229–233). Cairo, Egypt.
- Azad, A. P., & Chockalingam, A. (2006). Mobile base stations placement and energy aware routing in wireless sensor networks. In *IEEE wireless communications and networking conference* (pp. 264–269). Las Vegas, NV USA.
- Basagni, S., Carosi, A., Melachrinoudis, E., Petrioli, C., & Wang, Z. M. (2008). Controlled sink mobility for prolonging wireless sensor networks lifetime. *Wireless Networks*, 14(6), 831–858.
- Basagni, S., Carosi, A., Petrioli, C., & Phillips, C. A. (2011). Coordinated and controlled mobility of multiple sinks for maximizing the lifetime of wireless sensor networks. *Wireless Networks*, 17(3), 759–778.
- Behdani, B., Yun, Y. S., Smith, J. C., & Xia, Y. (2012). Decomposition algorithms for maximizing the lifetime of wireless sensor networks with mobile sinks. *Computing Operational Research*, 39(5), 1054–1061.
- Gandham, S. R., Dawande, M., Prakash, R., & Venkatesan, S. (2003). Energy efficient schemes for wireless sensor networks with multiple mobile base stations. In *IEEE global telecommunications conference* (pp. 377–381). San Francisco, USA.
- Gatzianas, M., & Georgiadis, L. (2008). A distributed algorithm for maximum lifetime routing in sensor networks with mobile sink. *IEEE Transactions on Wireless Communications*, 7(3), 984–994.
- Gu, Y., Bozdog, D., Ekici, E., Ozguner, F., & Lee, C. G. (2005). Partitioning based mobile element scheduling in wireless sensor networks. In *Second annual IEEE communications society conference on sensor and ad hoc communications and networks* (pp. 386–395).
- Guimarães, Dayan Adionel, Frigieri, Edielson Prevato, & Sakai, Lucas Jun (2020). Influence of node mobility, recharge, and path loss on the optimized lifetime of wireless rechargeable sensor networks. *Ad Hoc Networks*, 97, Article 102025.
- Güney, E., Aras, N., Altunel, İK., & Ersoy, C. (2010). Efficient integer programming formulations for optimum sink location and routing in heterogeneous wireless sensor networks. *Computer Networks*, 54(11), 1805–1822.
- Güney, E., Aras, N., Altunel, İK., & Ersoy, C. (2012). Efficient solution techniques for the integrated coverage, sink location and routing problem in wireless sensor networks. *Comps and Operational Research*, 39(7), 1530–1539.
- Jun, L., & Hubaux, J. P. (2007). Joint mobility and routing for lifetime elongation in wireless sensor networks. In *Annual joint conference of the IEEE computer and communications societies* (pp. 1735–1746). Miami, FL USA.

- Keskin, M. E. (2017). A column generation heuristic for optimal wireless sensor network design with mobile sinks. *European Journal of Operational Research*, 260(1), 291–304.
- Keskin, M. E., Altunel, İK., Aras, N., & Ersoy, C. (2011). Lifetime maximization in wireless sensor networks using a mobile sink with nonzero traveling time. *Computer Journal*, 54(12), 1987–1999.
- Li, J., & Mohapatra, P. (2007). Analytical modeling and mitigation techniques for the energy hole problem in sensor networks. *Pervasive Mobile Computing*, 3(3), 233–254.
- Liang, W., Luo, J., & Xu, X. (2010). Prolonging network lifetime via a controlled mobile sink in wireless sensor networks. In *Global telecommunications conference* (pp. 1–6).
- Luo, J., & Hubaux, J. P. (2010). Joint sink mobility and routing to maximize the lifetime of wireless sensor networks: the case of constrained mobility. *IEEE ACM Transactions Network*, 18(3), 871–884.
- Nesamony, S., Vairamuthu, M. K., Orłowska, M., & Sadiq, S. (2006). *On optimal route computation of mobile sink in a wireless sensor network technical report*. Queensland, Australia.
- Papadimitriou, I., & Georgiadis, L. (2005). Maximum lifetime routing to mobile sink in wireless sensor networks. (pp. 1–5). Split, Marina Frapa, Croatia: IEEE.
- Popa, L., Rostamizadeh, A., Karp, R., Papadimitriou, C., & Stoica, I. (2007). Balancing traffic load in wireless networks with curveball routing. In *International symposium on mobile ad hoc networking and computing* (pp. 170–179). Montréal, Québec, Canada: ACM.
- Somasundara, A. A., Ramamoorthy, A., & Srivastava, M. B. (2007). Mobile element scheduling with dynamic deadlines. *IEEE Transactions on Mobile Computing*, 6(4), 395–410.
- Türkoğulları, Y. B., Aras, N., Altunel, İK., & Ersoy, C. (2010a). A column generation based heuristic for sensor placement, activity scheduling and data routing in wireless sensor networks. *European Journal of Operational Research*, 207(2), 1014–1026.
- Türkoğulları, Y. B., Aras, N., & Altunel, İK. (2009). Optimal placement scheduling and routing to maximize lifetime in sensor networks. *Journal of Operational Research Society*, 61(6), 1000–1012.
- Türkoğulları, Y. B., Aras, N., Altunel, İK., & Ersoy, C. (2010b). An efficient heuristic for placement scheduling and routing in wireless sensor networks. *Ad Hoc Networks*, 8(6), 654–667.
- Vincze, Z., Fodor, K., Vida, R., & Vidács, A. (2006). Electrostatic modelling of multiple mobile sinks in wireless sensor networks. In *Proc. of the IFIP networking workshop on performance control in wireless sensor networks* (pp. 30–37). Coimbra, Portugal.
- Wang, Z. M., Basagni, S., Melachrinoudis, E., & Petrioli, C. (2005). Exploiting sink mobility for maximizing sensor networks lifetime. In *38th annual Hawaii international conference on system sciences* (p. 287a). Big Island, HI, USA: IEEE.
- Wu, X., Chen, G., & Das, S. K. (2008). Avoiding energy holes in wireless sensor networks with nonuniform node distribution. *IEEE Transactions on Parallel Distributing*, 19(5), 710–720.
- Yun, Y., & Xia, Y. (2010). Maximizing the lifetime of wireless sensor networks with mobile sink in delay-tolerant applications. *IEEE Transactions on Mobile Computing*, 9(9), 1308–1318.
- Yun, Y., Xia, Y., Behdani, B., & Smith, J. C. (2013). Distributed algorithm for lifetime maximization in a delay-tolerant wireless sensor network with a mobile sink. *IEEE Transactions on Mobile Computing*, 12(10), 1920–1930.